A Study on Migration and Immigration of Prey-Predator Ecology with Unlimited Resources

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ABSTRACT:

In this paper, we study the stability of two species ecology consisting of prey-predator with migration and immigration. The system comprises of a prey (S_1), a predator (S_2) that survives upon S_1 with immigration of prey and migration of predator. Here, both the species prey and predator have unlimited resources. The model equations of the system constitute a set of first order non-linear ordinary differential equations. All the possible equilibrium points are identified and the linearized equations for the perturbations over the points are analyzed to establish the criteria for stability. The system would be stable if all the characteristic roots were negative, in case they were real, and had negative real parts, in case they were complex.

Key words: Equilibrium State, Neutrally Stable, Predator, Prey, Trajectories, Unstable.

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1. INTRODUCTION:

Research in the area of theoretical ecology has been discussed by Gillman [3] and by Kot [4]. Several ecologists and mathematicians contributed to the growth of this area of knowledge. Mathematical ecology can be broadly divided into two main sub-divisions, Autecology and Synecology, which are described in the treatises of Anna Sher [1], Arumugam [2] and Sharma [21]. Syn-ecology is an ecosystem comprised of two or more distinct species. Species interact with each other in one way or the other. The Ecological interactions can be classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation, Parasitism and so on.

Mathematical Modeling plays a vital role in providing insight into the mutual relationships (positive, negative) between the interacting species. Several authors Ma [6], Moghadas [7], Murray [8] and Sze-Bi Hsu [23] introduced the general concepts of Modeling in Biological Science. Srinivas [22] studied the competitive ecosystem of two species and three species with limited and unlimited resources. Later, Narayan [9] studied prey-predator ecological models with partial cover for the prey and alternate food for the predator. Further, Kumar [5] studied some mathematical models of ecological commensalism. The present author Prasad [10-20] investigated continuous and discrete models on two, three and four species syn-ecosystems.

Predation is the interaction where one species gets benefits at the expense of the other. In this interaction between two organisms, one organism captures bio-mass from another. This is called a predator. In this interaction, one organism eats away at the other with its closeness of association. A real life example of predation is tiger and deer can be considered as predator and prey, that tiger feeds on other organisms and deer become a prey to the tiger.

2. NOTATION ADOPTED:

$N_1(t)$: The population strength of prey species (S_1)

$N_2(t)$: The population strength of predator species (S_2)

t : Time instant

a_i : Natural growth rates of S_i, $i = 1, 2$

a_{12}, a_{21} : Interaction coefficients of S_1 due to S_2 and S_2 due to S_1

M, I : Rate of migration and immigration of S_1 and S_2

Further the variables N_1, N_2 are non-negative and the model parameters a_1, a_2, a_{12}, a_{21}, M, I are assumed to be non-negative constants.

3. BASIC EQUATIONS OF THE MODEL:

The basic equation for the model is given by the following system of non-linear ordinary differential equations.

$$\frac{dN_1}{dt} = a_1 N_1 - a_{12} N_1 N_2 - MN_1; \ \frac{dN_2}{dt} = a_2 N_2 + a_{21} N_1 N_2 + IN_2 \qquad (1)$$

4. EQUILIBRIUM STATES:

The system under investigation has three equilibrium states given by $\frac{dN_1}{dt} = 0; \ \frac{dN_2}{dt} = 0$

$$E_1 : \bar{N}_1 = 0, \bar{N}_2 = 0; \ E_2 : \bar{N}_1 = -\left(\frac{a_2 + I}{a_{21}}\right), \bar{N}_2 = \left(\frac{a_1 - M}{a_{12}}\right), \text{ when } a_1 \neq M$$

$$E_{2*} : \bar{N}_1 = -\left(\frac{a_2 + I}{a_{21}}\right), \bar{N}_2 = 0, \text{ when } a_1 = M$$

5. STABILITY OF EQUILIBRIUM STATES:

Let $N_i(t) = (N_1, N_2) = \bar{N}_i + U_i(t); i = 1,2$

Where $U_i(t)$ is a small perturbation over the equilibrium point $N = (\bar{N}_1, \bar{N}_2)$. The basic equations are quasi lineralized to obtain the equations for the perturbed state as

$$\frac{du_1}{dt} = (a_1 - a_{12}\bar{N}_2 - M)u_1 - a_{12}\bar{N}_1 u_2; \ \frac{du_2}{dt} = a_{21}\bar{N}_2 u_1 + (a_2 + a_{21}\bar{N}_1 + I)u_2 \qquad (2)$$

The characteristic equation is $|A - \lambda I| = 0$ (3)

$$\text{Where } A = \begin{bmatrix} a_1 - a_{12}\bar{N}_2 - M & -a_{12}\bar{N}_1 \\ a_{21}\bar{N}_2 & a_2 + a_{21}\bar{N}_1 + I \end{bmatrix} \qquad (4)$$

The equilibrium state is stable, if all the roots of equation (3) are negative, in case they are real or have the negative real parts, in case they are complex.

5.1 Stability of fully washed out state (E_1):

The basic equations are linearized to obtain the equations as

$$\frac{du_1}{dt}=(a_1-M)u_1;\ \frac{du_2}{dt}=(a_2+I)u_2 \tag{5}$$

The characteristic equation of (5) is $\left[\lambda-(a_1-M)\right]\left[\lambda-(a_2+I)\right]=0$ (6)

The characteristic roots are (a_1-M) and (a_2+I).

Since one root is positive. Hence, E_1 is **unstable** and the solutions of the equations (5) are

$$u_1=u_{10}e^{(a_1-M)t};\ u_2=u_{20}e^{(a_2+I)t} \tag{7}$$

Where u_{10} and u_{20} are the initial values of u_1 and u_2 respectively.

Trajectories of perturbations:

The trajectories in u_1-u_2 plane are $\left(\frac{u_1}{u_{10}}\right)^{(a_2+I)}=\left(\frac{u_2}{u_{20}}\right)^{(a_1-M)}$

5.2 Stability of normal steady state (E_2):

In this state, the basic equations can be linearized, we get,

$$\frac{du_1}{dt}=a_{12}\left(\frac{a_2+I}{a_{21}}\right)u_2;\ \frac{du_2}{dt}=a_{21}\left(\frac{a_1-M}{a_{12}}\right)u_1,\ \text{when}\quad a_1\neq M \tag{8}$$

Characteristic equation is $\lambda^2-(a_2+I)(a_1-M)=0$ (9)

Let λ_1 and λ_2 be the roots of Characteristic equation (9)

Case (i): If $a_1>M$ then $\lambda_1>0$; $\lambda_2<0$

Therefore, E_2 is unstable.

Case (ii): When $a_1<M$, λ_1 and λ_2 are complex, therefore E_2 is neutrally stable.

The solutions curves are given by

$$u_1=\left[\frac{u_{20}-A\lambda_2u_{10}}{(\lambda_1-\lambda_2)A}\right]e^{\lambda_1t}+\left[\frac{A\lambda_1u_{10}-u_{20}}{(\lambda_1-\lambda_2)A}\right]e^{\lambda_2t};u_2=\lambda_1\left[\frac{u_{20}-A\lambda_2u_{10}}{(\lambda_1-\lambda_2)}\right]e^{\lambda_1t}+\lambda_2\left[\frac{A\lambda_1u_{10}-u_{20}}{(\lambda_1-\lambda_2)}\right]e^{\lambda_2t} \tag{10}$$

Where $A = \dfrac{a_{21}}{a_{12}(a_2 + I)} > 0$

Trajectories in $u_1 - u_2$ plane are, $u_2^2 = Bu_1^2 + 2c$

Where $B = \dfrac{a_{21}^2(a_1 - M)}{a_{12}^2(a_2 + I)}$ and c is arbitrary constant. $a_1 \neq M$

5.3 Stability of predator washed out state (E_{2*}) :

In this state, we get,

$$\frac{du_1}{dt} = a_{12}\left(\frac{a_2 + I}{a_{21}}\right)u_2; \; \frac{du_2}{dt} = 0 \tag{11}$$

The characteristic equation of (11) is $\lambda^2 = 0$

Hence, E_{2*} is neutrally stable, and the solutions of the equations (11) are

$$u_1 = u_{10} + a_{12}u_{20}\left(\frac{a_2 + I}{a_{21}}\right)t; \; u_2 = u_{20} \tag{12}$$

6. CONCLUSION:

The present paper deals with an investigation on the stability of two species a prey - predator eco-system with migration-immigration with unlimited resources. All possible equilibrium states of the model are identified and criteria for their stability are discussed. It is observed that,

i. The fully washed out state is unstable

ii. The normal steady state is neutrally stable when $a_1 < M$

iii. The predator washed out state is neutrally stable when $a_1 = M$

Further, the trajectories of solution curves for all equilibrium states are illustrated.

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