A Novel Fuzzy Arbitrary Order Predator-Prey Models

Divya Arumugam
Department of Mathematics and Actuarial Science
B.S.Abdur Rahman Crescent Institue of Science and Techonology, Chennai,India.
Email- divya_aru87@yahoo.com

P.S.Sheik Uduman
Department of Mathematics and Actuarial Science
B.S.Abdur Rahman Crescent Institute of Science and Technology, Chennai, India

Abstract- This paper describes A novel fuzzy arbitrary order predator-prey models..Lotka and Volterra was the first who introduced the predator-prey model which consists of two differential equations.The First equation represents the food species called prey and it is denoted by x(t) and second equation represent the feeding species predator which is denoted by y(t). Several researchers working in one prey and one predator model, Two-prey and one predator model, one prey and two predator model in different aspects. However, in this paper A Novel fuzzy arbitrary order predator-prey models represents one component is having two sort of Measurements. For example, the Blood Pressure (BP) is divided into lower blood pressure and high blood pressure. So this model is helpful to find the interaction between the predator and prey and also predict the time in fractional order. The Proposed model is very effective in the field of medicine.

Keywords – Predator-Prey Model, Homotopy Perturbation Method, New models of fuzzy Arbitrary Order predator-prey equation, two sort of Measurements.

I. Introduction

Lotka and Voltera introduced the mathematical model of predator-prey equation. Seeks the interaction of predator as well as prey. Later many researchers worked on one prey and one predator model, Two prey and one predator model, and one prey and two predator model in different aspects. But in this paper the new model of fuzzy arbitrary order predator-prey equation represents one predator having two sort of measurements and one prey having two sort of Measurements. Here the two sort of Measurements represents the one risk factor having two bounds, i.e) lower bound and upper bound. So this model helps to find the interaction between the predator and prey having two sort of Measurements

A Gauss type general prey-predator model is proposed and analyzed to study the effect of predation on two competing prey species and they observed the intra-specific interference factors governing the dynamics of the system[1] Single prey and predator population model was discussed and they can be separated by reproduction ability into an immature and mature stage [2]. The global stability of diffusive predator-prey system of holling -tanner type in a bounded domain [3]. A time delayed reaction -diffusion system with homogenous neumann boundary conditions. In this system they describes two predators competing for the same prey by upper and lower solutions [4].Lotka and Voltera first introduced a mathematical model named as lotka and voltera model. Later increasing number of publication on the subject of bio-mathematics. Mathematical modeling is currently playing an important role in the study of both theoretical and applied biology [5]. Lotka-Volterra dynamics with two competing spices which are affected not only by harvesting but also by the presence of a predator, the third species [6]. Dynamics of predator-prey models has been discussed by a lot of papers. It is well known in many applications, the nature of the permanence and global asymptotic stability of predator-prey models is of great interest [7] Two-Prey and One-Predator interaction and One-Prey and Two-Predator interaction. The global properties of the classical three dimensional lotka two-prey and one-predator model and one-prey two-predator systems. One species is always driven to extinction and the system behaves asymptotically as a two-dimensional Predator-Prey Lotka-Voltera system [8]. A mathematical model is proposed and analyzed to study the dynamics of one-prey and two-predators system with ratio-dependent predators growth rate [9]. One Prey Species living in two different habitats and a predator where a prey exhibits group defense is studied [10]. The dynamical interactions of three species food chain model is presented where two predators competing on one prey [11]. But in this paper we discussed about one kind of prey is having two sorts of measurements and one kind of predator is having two sorts of measurements with different kind of model was proposed for the analyzation.

II. Proposed Model

2.1 New Fuzzy Arbitrary order Predator-Prey Model

A New fuzzy arbitrary order predator-prey model is derived in this chapter and it is the effective tool to predict the time in fractional-order using HPM. Compared to the Lotka-Volterra model these new models are more efficient. These new models are applicable for finding the time in fractional when a predator and prey interact with each other. It is helpful to diagnose the disease quickly, according to the fractional order time derivative and this model is applicable only for some particular cases

2.2 Fuzzy arbitrary order Predator-Prey Model-1

S.Das and P.K.Gupta [13] introduced the solution of fractional order time derivatives of Lotka-Voltera equation with the help of Homotopy Perturbation Method [HPM] which performs extremely well regarding efficiency and simplicity to solve the mathematical model and it is given below

$$D_t^{\alpha}\tilde{x}(t) = a\tilde{x}(t) - b\tilde{x}(t)\tilde{y}(t) \quad (1)$$

$$D_t^{\beta}\tilde{y}(t) = c\tilde{x}(t)\tilde{y}(t) - d\,\tilde{y}(t) \quad (2)$$

Where D_t^{α} and D_t^{β} are the Caputo derivatives of order $0 < \alpha, \beta \leq 1$,'**a**' represents prey's birth/growth rate,'**b**' represents the predators rate of consumption of prey,'**c**' represents predators growth rate as a result of feeding on prey, '**d** 'denotes death rate of predator, and also **a,b,c,** and **d** as functions of time '**t'**.Therefore the related predator-prey system is given in the equation 3& 4

$$D_t^{\alpha}\tilde{x}(t) = a(t)\tilde{x}(t) - b(t)\tilde{x}(t)\tilde{y}(t) \quad (3)$$

$$D_t^{\beta}\tilde{y}(t) = c(t)\tilde{x}(t)\tilde{y}(t) - d(t)\tilde{y}(t) \quad (4)$$

2.2 Fuzzy arbitrary order Predator-Prey Model-2

Zhouji- Cui and Zuodong yang [14] introduced the solutions of fractional Lotka-Volterra equation with fractional time derivatives using HPM. The derived equation is given

$$D_t^{\alpha}\tilde{x}(t) = a\tilde{x}(t) - b\tilde{x}(t)\tilde{y}(t) \quad (5)$$

$$D_t^{\beta}\tilde{y}(t) = c\tilde{x}(t)\tilde{y}(t) - d\tilde{y}(t) + e\tilde{y}^2(t) \quad (6)$$

Where D_t^{α} and D_t^{β} are the Caputo derivative of orders **$0<\alpha, \beta \leq 1$.** In this study let us consider '**a**' represents prey's birth/growth rate,'**b**' represents the predators rate of consumption of prey,'**c**' represents predators growth rate as a result of feeding on prey'**d** 'denotes death rate of predator, '**e**' denotes predators growth rate of one kind predator is having two sort of Measurements in fractional order and also **a,b,c,d,**and **e** as functions of time '**t'**. Therefore the related predator-prey system is given in the equation

$$D_t^{\alpha}\tilde{x}(t) = a(t)\tilde{x}(t) - b(t)\tilde{x}(t)\tilde{y}(t) \quad (7)$$

$$D_t^{\beta}\tilde{y}(t) = c(t)\tilde{x}(t)\tilde{y}(t) - d(t)\tilde{y}(t) + e(t)\tilde{y}^2(t) \quad (8)$$

2.3 Fuzzy arbitrary order Predator-Prey Model-3

New fuzzy arbitrary order predator-prey model 1 is developed based on the model-1

$$D_t^{\alpha}\tilde{x}(t) = a\tilde{x}(t) - b\tilde{x}(t)\tilde{y}(t) \quad (9)$$

$$D_t^{\beta}\tilde{y}(t) = c\tilde{x}(t)\tilde{y}(t) - d\tilde{y}(t) + e\tilde{y}^2(t) - f\,\tilde{x}^2(t)\tilde{y}(t) \quad (10)$$

Where D_t^{α} and D_t^{β} are the caputo derivative of orders **0<**$\alpha, \beta \leq 1$. In this study let us consider '**a**' represents prey's birth/growth rate,'**b**' represents predators rate of consumption of prey,'**c**' represents predators growth rate as a result of feeding on prey'**d** 'denotes death rate of predator, '**e**' denotes predators growth rate of one kind predator is having two sort of Measurements, '**f** 'denotes the prey death rate when one kind of prey is having two sort of Measurements as a rate of consumption of predator result is given in the fractional order and also **a, b, c, d, e,** and **f** as functions of time '**t'**. Therefore the related predator-prey system is given in the equation .

$$D_t^{\alpha}\tilde{x}(t) = a(t)\tilde{x}(t) - b(t)\tilde{x}(t)\tilde{y}(t) \quad (11)$$

$$D_t^{\beta}\tilde{y}(t) = c(t)\tilde{x}(t)\tilde{y}(t) - d(t)\tilde{y}(t) + e(t)\tilde{y}^2(t) - f(t)\tilde{x}^2(t)\tilde{y}(t) \quad (12)$$

2.4 Fuzzy arbitrary order Predator-Prey Model- 4

New fuzzy arbitrary order predator-prey model 2 is developed based on the model-1

$$D_t^{\alpha}\tilde{x}(t) = a\,\tilde{x}(t) - b\,\tilde{x}(t)\tilde{y}(t) \quad (13)$$

$$D_t^{\beta}\tilde{y}(t) = c\,\tilde{x}(t)\tilde{y}(t) - d\,\tilde{y}(t) + e\,\tilde{y}^2(t) + f\,\tilde{x}^2(t)\tilde{y}(t) \quad (14)$$

Where D_t^{α} and D_t^{β} are the caputo derivative of orders **0<**$\alpha, \beta \leq 1$. In this study considered '**a**' represents prey's birth/growth rate,'**b**' represents predators rate of consumption of prey,'**c**' represents predators growth rate as a result of feeding on prey'**d** 'denotes death rate of predator, '**e**' denotes predators growth rate of one kind predator is having two sort of Measurements, '**f'** denotes the prey growth rate when one kind of prey is having two sort of Measurements as a rate of consumption of predator result is given in fractional order furthermore **a, b, c, d,e,** and **f** as functions of time '**t'**. Hence the related predator-prey system is given in the equation 15 &16

$$D_t^{\alpha}\tilde{x}(t) = a(t)\tilde{x}(t) - b(t)\tilde{x}(t)\tilde{y}(t) \quad (15)$$

$$D_t^{\beta}\tilde{y}(t) = c(t)\tilde{x}(t)\tilde{y}(t) - d(t)\tilde{y}(t) + e(t)\tilde{y}^2(t) + f(t)\tilde{x}^2(t)\tilde{y}(t) \quad (16)$$

2.5 Fuzzy arbitrary order predator-prey model-5

New fuzzy arbitrary order predator-prey model 3 is developed based on the model which is in the given below

$$D_t^{\alpha}\widetilde{x}(t) = a\widetilde{x}(t) - b\widetilde{x}(t)\widetilde{y}(t) \tag{17}$$

$$D_t^{\beta}\widetilde{y}(t) = c\widetilde{x}(t)\widetilde{y}(t) - d\widetilde{y}(t) + e\widetilde{y}^2(t) + f\widetilde{x}^2(t)\widetilde{y}^2(t) \tag{18}$$

Where D_t^{α} and D_t^{β} are the caputo derivative of orders $0<\alpha, \beta \leq 1$. In this study we considered '**a**' represents prey's birth/growth rate, '**b**' represents predators rate of consumption of prey,'**c**' represents predators growth rate as a result of feeding on prey '**d** 'denotes death rate of predator, '**e**' denotes predators growth rate of one kind predator is having two sort of Measurements, '**f**' denotes predators growth rate as a result of feeding on prey when one kind of prey and predator is having two sort of Measurements result in fractional order furthermore **a, b, c, d, e,** and **f** as functions of time '**t**'. Therefore the related predator-prey system is given in the equation

$$D_t^{\alpha}\widetilde{x}(t) = a(t)\widetilde{x}(t) - b(t)\widetilde{x}(t)\widetilde{y}(t) \tag{19}$$

$$D_t^{\beta}\widetilde{y}(t) = c(t)\widetilde{x}(t)\widetilde{y}(t) - d(t)\widetilde{y}(t) + e(t)\widetilde{y}^2(t) + f(t)\widetilde{x}^2(t)\widetilde{y}^2(t) \tag{20}$$

2.6 Fuzzy arbitrary order Predator-Prey Model-6

New fuzzy arbitrary order predator-prey model 4 is developed based on the model -1

$$D_t^{\alpha}\widetilde{x}(t) = a\widetilde{x}(t) - b\widetilde{x}(t)\widetilde{y}(t) \tag{21}$$

$$D_t^{\beta}\widetilde{y}(t) = c\widetilde{x}(t)\widetilde{y}(t) - d\widetilde{y}(t) + e\widetilde{y}^2(t) - f\widetilde{x}^2(t)\widetilde{y}^2(t) \tag{22}$$

Where D_t^{α} and D_t^{β} are the caputo derivative of orders $0<\alpha, \beta \leq 1$. In this study we considered '**a**' represents prey's birth/growth rate, '**b**' represents predators rate of consumption of prey, '**c**' represents predators growth rate as a result of feeding on prey '**d** 'denotes death rate of predators, '**e**' denotes predators growth rate of one kind predator is having two sort of Measurements, '**f**' denotes predators death rate as a result of feeding on prey when one kind of prey and predator is having two sort of Measurements result in fractional order furthermore **a, b, c, d, e,** and **f** as functions of time '**t**'. Therefore the related predator-prey system is given in the equation

$$D_t^{\alpha}\widetilde{x}(t) = a(t)\widetilde{x}(t) - b(t)\widetilde{x}(t)\widetilde{y}(t) \tag{23}$$

$$D_t^{\beta}\widetilde{y}(t) = c(t)\widetilde{x}(t)\widetilde{y}(t) - d(t)\widetilde{y}(t) + e(t)\widetilde{y}^2(t) - f(t)\widetilde{x}^2(t)\widetilde{y}^2(t) \tag{24}$$

2.7 Fuzzy arbitrary order Predator-Prey Model-7

New fuzzy arbitrary order predator-prey model 5 is developed based on the model-1

$$D_t^{\alpha}\tilde{x}(t) = a\tilde{x}(t) - b\tilde{x}(t)\tilde{y}(t) \tag{25}$$

$$D_t^{\beta}\tilde{y}(t) = c\tilde{x}(t)\tilde{y}(t) - d\tilde{y}(t) + e\tilde{y}^2(t) - f\tilde{x}^2(t)\tilde{y}(t) + g\tilde{x}^2(t)\tilde{y}^2(t) \tag{26}$$

Where $\boldsymbol{D_t^{\alpha}}$ and $\boldsymbol{D_t^{\beta}}$ are the caputo derivative of orders **0<**$\boldsymbol{\alpha}$ **,**$\boldsymbol{\beta \leq 1}$. In this study we considered '**a**' represents prey's birth/growth rate, '**b**' represents predators rate of consumption of prey, '**c**' represents predators growth rate as a result of feeding on prey '**d** 'denotes death rate, '**e**' denotes predators growth rate of one kind predator is having two sub-parts, '**f**' denotes the prey death rate when one kind of prey is having two sort of Measurements as a rate of consumption of predator. '**g**' denotes predators growth rate as a result of feeding on prey when one kind of prey and predator is having two sort of Measurements result in fractional order furthermore **a, b, c, d, e, f** and **g** as functions of time '**t**'. Therefore the related predator-prey system is now defined as.

$$D_t^{\alpha}\tilde{x}(t) = a(t)\tilde{x}(t) - b(t)\tilde{x}(t)\tilde{y}(t) \tag{27}$$

$$D_t^{\beta}\tilde{y}(t) = c(t)\tilde{x}(t)\tilde{y}(t) - d(t)\tilde{y}(t) + e(t)\tilde{y}^2(t) - f(t)\tilde{x}^2(t)\tilde{y}(t) + g(t)\tilde{x}^2(t)\tilde{y}^2(t) \tag{28}$$

2.8 Fuzzy arbitrary order Predator-Prey Model-8

New fuzzy arbitrary order predator-prey model 6 is developed based on the model-1

$$D_t^{\alpha}\tilde{x}(t) = a\tilde{x}(t) - b\tilde{x}(t)\tilde{y}(t) \tag{29}$$

$$D_t^{\beta}\tilde{x}(t) = c\tilde{x}(t)\tilde{y}(t) - d\tilde{y}(t) + e\tilde{y}^2(t) - f\tilde{x}^2(t)\tilde{y}(t) - g\tilde{x}^2(t)\tilde{y}^2(t) \tag{30}$$

Where D_t^{α} and D_t^{β} are the caputo derivative of orders 0<α ,$\beta \leq 1$. In this study we considered **'a'** represents prey's birth/growth rate, '**b**' represents predators rate of consumption of prey, **'c'** represents predators growth rate as a result of feeding on prey, '**d** 'denotes death rate of predator, **'e'** denotes predators growth rate of one kind predator is having two sort of Measurements, **'f'** denotes the prey growth rate when one kind of prey is having two sort of Measurements as a rate of consumption of predator **'g'** denotes predators death rate as a result of feeding on prey when one kind of prey and predator is having two sort of Measurements result in fractional order together with **a, b, c, d, e, f** and **g** as functions of time. Therefore the related predator-prey system is given in the equation

$$D_t^{\alpha}\tilde{x}(t) = a(t)\tilde{x}(t) - b(t)\tilde{x}(t)\tilde{y}(t) \tag{31}$$

$$D_t^{\beta}\tilde{x}(t) = c(t)\tilde{x}(t)\tilde{y}(t) - d(t)\tilde{y}(t) + e(t)\tilde{y}^2(t) - f(t)\tilde{x}^2(t)\tilde{y}(t) - g(t)\tilde{x}^2(t)\tilde{y}^2(t) \tag{32}$$

III.CONCLUSION

In this paper we successfully introduced the new models of fuzzy arbitrary order predator-prey model. In future we plan to validate this model with the help of Expert's.

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