NEW SETS AND TOPOLOGIES IN FUZZY IDEAL TOPOLOGICAL SPACES

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Abstract

The purpose of this paper is to introduce fuzzy L_I-perfect, fuzzy R_I-perfect, fuzzy C_I-perfect in fuzzy ideal topological spaces. Furthermore, we have introduced R_I topology using fuzzy ideal topological spaces. The characterization for compatible ideals via fuzzy R_I-perfect sets and a fuzzy topology via ideals which is finer than τ using fuzzy R_I -perfect sets on a finite set is obtained. Moreover, the fuzzy ∧∗set is introduced. Some related concepts like fuzzy ∨∗ set and fuzzy$^{*}{}_{s}$– connected set are also defined. The characterizations along with their properties concerning all these concepts are discussed.

Keywords: *fuzzy L_I-perfect, fuzzy R_I-perfect, fuzzy C_I-perfect, fuzzy∧∗set, fuzzy ∨∗ set, fuzzy$^{\star}{}_{s}$ – connected set, R_I-topology.*

1.Introduction and preliminaries

The study of space and continuity in mathematics is concerned with topology. Since it's included in theresearch of continuous deformations of a space, it is also referred to as rubber sheet geometry for the study of continuous deformations of a room. During the 19th century, the term topology was introduced by Johann Benedict. Fuzzy is one of the most important and valuable thoughts in the contemporary scientific world. Zadeh [1] first contributed his work to the introduction of fuzzy sets in 1965. In Vaidyanathaswamy[2]introduced his concept in 1945 to introduce ideal topological space.

In 1990, Jankovic and Hamlett [3] defined the concept of the I-open set in ideal topological space.Mahmoud [4] and Sarkar [5] proposed some of the ideal concepts in the fuzzy pattern and Sarkar separately in 1997.Many other properties have been explored. Fuzzy continuity decomposition is one of the many issues in fuzzy topology. When decomposition is done via fuzzy topological ideals, it becomes very interesting.

X represents a non-empty fuzzy set and fuzzy subset A of X, denoted by A≤ X, is defined in this article. In the context of Zadeh, via a membership function.The empty set, the entire set and the class are the fundamental fuzzy sets of all fuzzy subsets of X, which are denoted respectively by 0, 1, and I^X. The topology of a subfamily I^X would denote the topology of fuzzy sets on I^X as described by Chang[6].

By (x τ), we mean, in Chang's sense, a fuzzy topological space. A fuzzy point X with the support x∈X and the value $\alpha(0 < \alpha \leq 1)$ is denoted by x_α.Cl(A), Int(A) and 1-A for a fuzzy subset A of X, the fuzzy closure, fuzzy interior and fuzzy complement of A will, respectively, denote. A non-empty Set I of fuzzy ideal is called fuzzy subsets of X if and only if

1. B ∈ I and A ≤ B, then A ∈ I (heredity),

2. if A ∈ I and B ∈ I then A v B ∈ I (finite additivity).

The triple (X, τ, I) means a fuzzy topological space with an ideal I fuzzy and topology τ fuzzy. For (x, τ, I), the fuzzy local function of A ≤ X with respect to τ and I is denoted by A* (τ, I) (briefly A*) and is defined as

A* (τ, I) = Λ{x ∈ X: A Λ U ∉ I for every U ∈ τ(x)}. While A* is the union of the fuzzy points x, if U ∈ τ(x) and E ϵ I, then there is a union of the fuzzy points x. A minimum of one y ∈ x for which U(y) + A(y) − 1 > E (y). Fuzzy closure operator of a fuzzy set in (X, τ, I) and it is defined as Cl* (A) = A ∨ A *. The set in (X, τ, I) implies an extension of Fuzzy topological space over the fuzzy ideal in (X, τ, I), which is constructed as a basis[7] by considering the class β = {U − E: U ∈ τ, E∈ I}.This Fuzzy Set Topology is the generalization of the ordinary one is considered.

First, let us recall some definitions which are used in the sequel.

Definition 1.1. A subset of A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy τ ∗closed if A= Cl∗(A).

Definition 1.2. A fuzzy subset A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy regular-I– closed if A = (Int(A)) ∗.

Lemma 1.1. For a subset A of a fuzzy topological space (X, τ), the following property holds:

(a) sCl(A) = A ∨ Int(Cl(A)),

(b) sCl(A) = Int(Cl(A)) if A is fuzzy open.

Lemma 1.2. Let (X, τ) be an ideal topological space with an arbitrary index ∆, I an ideal of subsets of X and ρ(X) the power set of X. If {Aα: α ∈ ∆} ≤ ρ(X), then the following property holds: ∨α∈∆ (A ∗ α) ≤(∨α∈∆Aα) ∗

Definition 1.3A subset A of a fuzzy ideal topological space (X, τ, I) is said to be

fuzzy α- I-open if A ≤ Int(Cl*(Int(A))),

fuzzy semi-I-open if A ≤ Cl*(Int(A)),

fuzzy pre- I -open if A ≤ Int (Cl*(A)),

fuzzy β - I-open if A ≤ Cl (Int (Cl*(A))),

fuzzy δ- I -open if Int (Cl*(A)) ≤ Cl*(Int(A)),

fuzzy strong β - I -open if A ≤ Cl*(Int (Cl*(A))).

The complement of fuzzy α - I –open (resp. fuzzy semi- I -open, fuzzy pre- I -open, fuzzy β - I -open, fuzzy strong β - I-open) is fuzzy α - I -closed (resp. fuzzy semi- I - closed, fuzzy pre- I -closed, fuzzy β- I -closed, fuzzy strong β - I -closed).

The family of all fuzzy α - I-open (resp. fuzzy semi- I -open, fuzzy pre- I -open, fuzzy β-I -open, fuzzy strong β - I-open, fuzzy δ -I-open) sets in (X, τ, I) will be denoted by FαIO(X) (resp. F SIO(X), F P IO(X), F βIO(X), F SβIO(X), F δIO(X)).

2.. Fuzzy L_I-perfect, Fuzzy R_I-perfect, Fuzzy C_I-perfect

Definition 2.1. Let (X, τ, I) be a fuzzy ideal topological space. A subset A of X is said to be [8]

(a) fuzzy L_I-perfect if A − A∗∈I,

(b) fuzzy R_I-perfect if A∗ − A ∈I,

(c) fuzzy C_I-perfect if A is both fuzzy L_I-perfect and fuzzy R_I-perfect.

The collection of fuzzy L_I -perfect sets, fuzzy R_I -perfect sets and fuzzy C_I -perfect sets in (X, τ, I) is denoted by L, R and C.

Proposition 2.1. If a subset A of an fuzzy ideal topological space (X, τ, I) is fuzzy C_I-perfect, then A∆A∗∈I.

Proof:

Since A ∈I, A∗= φ. Then A − A∗= A ∈ I and A∗ − A = φ ∈ I. Hence A is both a fuzzy L_I-perfect and fuzzy R_I-perfect set.

Proposition 2.2. In a fuzzy ideal topological space (X, τ, I), every fuzzy τ ∗-closed set is fuzzy $R_I$perfect.

Proof:

Let Abe a fuzzy τ ∗-closed set. Therefore, A∗⊆ A. Hence A∗ − A = φ =I. Therefore, A is a fuzzy R_I- perfect set.

Corollary 2.1. In a fuzzy ideal topological space (X, τ, I),

(a) 0 and 1 are fuzzy R_I-perfect sets

(b) For any fuzzy subset of A of an fuzzy ideal topological space (X,τ,I),cl(A), A∗, cl∗(A) are fuzzy $R_I$perfect sets

(c) Every fuzzy regular- I -closet is fuzzy R_I -perfect.

Proof:

The proof follows from proposition 2.2.

Proposition 2.3. If a fuzzy subset A of a fuzzy ideal topological space (X, τ, I) is such that A ∈I, then A is fuzzy C_I-perfect.

Proof: Since A ∈I, A∗= φ. Then A − A∗= A ∈I and A∗ − A = φ ∈I. Hence A is both a fuzzy L_I -perfect and fuzzy R_I -perfect set.

Corollary 2.2. Let A be a fuzzy subset of a fuzzy ideal topological space (X, τ, I).Consider the following.

(a) If A ∈I, then every fuzzy subset of A is a fuzzy C_I-perfect set.

(b) If A is fuzzy R_I-perfect, then A∗−A is fuzzy C_I-perfect.

(c) If A is fuzzy L_I-perfect set, then A − A∗ is a fuzzy C_I-perfect set.

(d) If A is fuzzy C_I-perfect, then AΔA∗ is a fuzzy C_I-perfect set.

Proof :

The proof follows from proposition 2.3.

Proposition 2.4. In a fuzzy ideal topological space (X, τ, I), every fuzzy *-dense-in- itself is a fuzzy L_I - perfect set.

Proof :

Let A be a fuzzy *-dense-in-itself set of X. Then A ⊆A∗. Therefore, A − A∗= φ ∈I. Hence A is fuzzy L_I perfect set.

Corollary 2.3. In a fuzzy ideal topological space (X, τ, I),

(a) every fuzzy I -open set is fuzzy L_I-perfect set,

(b) every fuzzy almost I -open set is fuzzy L_I-perfect set,

(c) every fuzzy regular - I -closed set is fuzzy L_I-perfect,

Proof:

Since all the above sets are fuzzy *-dense-in-itself, by proposition 2.4, these sets are fuzzy L_I-perfect.

Remark 2.1.

The members of the fuzzy ideal of an fuzzy ideal space are fuzzy L_I perfect, but the non-empty members of the fuzzy ideal are not fuzzy *-dense-itself. Therefore, the converse of the above corollary and proposition 2.4 need not to be true.

Proposition 2.5. In a fuzzy ideal topological space (X, τ, I),

(a) empty set is an fuzzy L_I-perfect set,

(b) X is a fuzzy L_I-perfect set if the fuzzy ideal is codense.

Proof :

(a) Since φ − φ∗= φ ∈I, the empty set is an L_I -perfect set.

(b) We know that X = X∗ if and only if the fuzzy ideal I is codense. Then X − X∗= φ ∈I. Hence the result follows.

Proposition 2.6. Let (X, τ, I) be a fuzzy ideal topological space. Let A and B be two subsets of X such that A ⊆ B and A∗= B∗, then

(a) B is fuzzy R_I-perfect if A is fuzzy R_I-perfect.

(b) A is fuzzy L_I-perfect if B is fuzzy L_I-perfect.

Proof:

(a) Let A be a fuzzy R_I -perfect set. Then A∗ − A ∈I. Now, B∗−B = A∗−B ⊆ A∗−A. By heredity property of ideals, B∗−B ∈ I. Hence B is fuzzy R_I-perfect.

(b) Let B be a fuzzy L_I -perfect set. Then B − B∗∈I. Now, A − A∗=A − B∗⊆B -B∗. By heredity property of ideals, A − A∗∈I. Hence A is fuzzy L_I-perfect.

Corollary 2.4. Let (X, τ, I) be a fuzzy ideal topological space. Let A and B be two subsets of X such that A ⊆ B ⊆cl∗A, then

(a) B is fuzzy R_I-perfect if A is fuzzy R_I-perfect.

(b) A is fuzzy L_I-perfect if B is fuzzy L_I-perfect.

Proof :

Since A ⊆ B ⊆cl∗A; A∗⊆ B∗⊆ (cl∗A)∗ = A∗. Hence A∗ = B∗. Therefore, the result follows form proposition 2.6.

Proposition 2.7. Let A be a subset of an fuzzy ideal topological space (X, τ, I) such that A is fuzzy L_I- perfect set and A ∧ A∗ is R_I-perfect; then both A and A ∧ A∗ are fuzzy C_I-perfect.

Proof:

Since A is fuzzy L_I-perfect, A − A∗∈I. By the condition for every I ∈I,(A ∨I)∗= A∗= (A −I)∗. Therefore, (A ∨ (A − A∗)) ∗= A∗= (A − (A −A∗)) ∗. This implies A∗= (A ∧A∗) ∗. Therefore, we have A ∧ A∗⊆ A with (A ∧A∗) ∗. By Proposition 2.6, A is fuzzy R_I-perfect if A∧ A∗ is fuzzy R_I-perfect and A ∧A∗ is fuzzy L_I -perfect if A is fuzzy L_I -perfect set. Hence A is fuzzy R_I -perfect and A ∧A∗ is fuzzy L_I -perfect.

Proposition 2.8. If a subset A of a fuzzy ideal topological space (X, τ, I) is fuzzy R_I -perfect set and A∗ is fuzzy L_I -perfect, then A ∧ A∗ is fuzzy L_I -perfect.

Proof:

Since A is fuzzy R_I -perfect, A∗ − A ∈ I. By the condition for every I ∈I, (A∨I)∗ = A∗ = (A−I)∗. Therefore, (A∗∨(A∗−A))∗ = A∗∗ = (A∗−(A∗−A))∗. This implies A∗∗ = (A ∧A∗) ∗. Therefore, we have A ∧A∗⊆ A∗ with (A ∧A∗)∗ = A∗∗. By Proposition 2.6, A ∧A∗ is fuzzy L_I -perfect if A∗ is fuzzy L_I -perfect set. Hence A ∧A∗ is fuzzy L_I-perfect.

Proposition 2.9. If A and B are fuzzy R_I -perfect sets, then A ∨B is a fuzzy R_I -perfect set.

Proof:

Let A and B be fuzzy R_I -perfect sets. Then A∗ − A ∈I and B∗− B ∈I. By finite additive property of fuzzy ideals, (A∗− A) ∨ (B∗ −B) ∈I. Since (A∗∨B∗) − (A∨B) ⊆ (A∗−A) ∨ (B∗−B), by heredity property (A∗∨B∗) − (A∨B) ∈I. Hence (A ∨B) ∗− (A ∨B) ∈I. This proves the result.

Corollary 2.5. Finite union of fuzzy R_I -perfect sets is a fuzzy R_I-perfect set.

Proof:

The proof follows from Proposition 2.9.

Proposition 2.10. If A and B are fuzzy L_I -perfect sets, then A ∨B is a fuzzy L_I -perfect set.

Proof: Since A and B be fuzzy L_I -perfect sets. Then A − A∗∈ Iand B − B∗∈ I. Hence by finite additive property of fuzzy ideals, (A − A∗) ∨ (B − B∗) ∈ I. Since (A∨B) − (A∨B) ∗ = (A∨B)− (A∗∨B∗) ⊆ (A−A∗) ∨ (B−B∗), by heredity property (A ∨B) − (A ∨B) ∗∈I. This proves that A ∨ B is a fuzzy L_I -perfect set.

Corollary 2.6. Finite union of fuzzy L_I -perfect sets is a fuzzy L_I -perfect sets.

Proof:

The proof follows from Proposition 2.10.

Proposition 2.11. If A and B are fuzzy R_I -perfect sets, then A ∧B is a fuzzy R_I -perfect set.

Proof:

Suppose that A and B be fuzzy R_I -perfect sets. Then A∗ − A ∈I and B∗− B ∈I. By finite additive property of fuzzy ideals, (A∗−A) ∨ (B∗−B) ∈ I. Since (A∗∧B∗) − (A∧B) ⊆ (A∗−A) ∨ (B∗−B), by heredity property (A∗∧B∗) − (A∧B)∈ I. Also (A ∧B)∗ − (A ∧B) ⊆ (A∗∧B∗) − (A ∧B) ∈I. This proves the result.

Corollary 2.7.

Finite intersection of fuzzy R_I -perfect sets is a fuzzy R_I -perfect set.

Proof:

The proof follows from Proposition 2.11.

Proposition 2.12. Finite union of fuzzy C_I -perfect sets is a fuzzy C_I -perfect set.

Proof:

The proof follows from Corollaries 2.6 and 2.7, finite union of fuzzy C_I-perfect sets is a fuzzy $C_I$perfect set.

Proposition 2.13. If (X, τ, I) is a fuzzy ideal topological space with X being finite, then the collection R is a fuzzy topology which is finer than the topology of fuzzy τ ∗-closed sets.

Proof:

By Corollary 2.1, 0 and 1 are fuzzy R_I -perfect sets. By Corollary 2.5, finiteunion of fuzzy R_I perfect setsisa fuzzy R_I -perfect sets, and by Corollary 2.7, finite intersection of fuzzy R_I -perfect sets is fuzzy R_I - perfect. Hence the collection R is a fuzzy topology if X is finite. Also, by Proposition 2.2 every fuzzy τ ∗-closed set is a fuzzy R_I -perfect set. Hence the fuzzy topology R is finer than the topology of fuzzy τ ∗- closed sets if X is finite.

Proposition 2.14. In a fuzzy ideal topological space (X, τ, I), (fuzzy τ ∗ -closed sets) ∨ I ⊆R.

Proof:

The proof follows from Propositions 2.2 and 2.3. The following example shows that (fuzzy τ ∗closed set) ∨I ⊆ R.

Example 2.1. Let (X, τ, I) be a fuzzy ideal topological space with X = (a, b, c) and A, B, and C be fuzzy subsets of X defined as follows:

A(a)=0.3, A(b)=0.5,A(c)=0.8

B(a)=0.2, B(b)=0.6,B(c)=0.9

C(a)=0.5, C(b)=0.7,C(c)=0.2

we have τ = {0, A, 1}. if we take I = ρ(x), then the collection of (fuzzy τ ∗-closed sets) is A = cl∗A and R =0. Now (fuzzy τ ∗-closed sets) ∨I = A f= R.

Proposition 2.15. Let (X, τ, I) be a fuzzy ideal topological space and A ⊆X. The set A is fuzzy R_I-perfect if and only if F ⊆ A∗ − A in X implies that F ∈I

Proof:

Assume that A is fuzzy R_I -perfect set. Then A∗ − A ∈ I. By heredity property of fuzzy ideals, every set F⊆ A∗ − A in X is also in I. Conversely assume that F ⊆ A∗ − A in X implies that F ∈I. Since A∗ − A is a subset of itself, by assumption A∗ − A ∈I. Hence A is fuzzy R_I -perfect.

Proposition 2.16. Let (X, τ, I) be a fuzzy ideal topological space and A ⊆X. The set A is fuzzy L_I-perfect if and only if F ⊆ A − A∗ in X implies that F ∈I.

Proof:

Assume that A is fuzzy L_I -perfect set. Then A − A∗∈I. By heredity property of fuzzy ideals, every set F ⊆ A − A∗ in X is also in I. Conversely assume that F ⊆ A − A∗ in X implies that F ∈I. Since A − A∗ is a subset of itself, by assumption A − A∗∈I. Hence A is fuzzy L_I -perfect.

Proposition 2.17. Let (X, τ) be a fuzzy topological space and A ⊆X. Let I_1 and I_2 be two ideals on X with I_1⊆I_2. Then A is fuzzy R_I-perfect with respect to I_2 if it is fuzzy R_I-perfect with respect to I_1.

Proof :

Since I_1⊆I_2, A∗(I_2) ⊆ A∗(I_1). Let A be fuzzy R_I-perfect with respect to I_1. Then A∗(I_1) − A∈I_1. Also, A∗(I_2) − A ⊆ A∗(I_1) − A. Hence by heredity property of fuzzy ideals, A∗(I_2) − A∈I_1⊆I_2.Therefore A is fuzzy R_I- perfect with respect toI_2.

Theorem 2.1. Let (X, τ) be a fuzzy topological space with an fuzzy ideal I on X . Then the following are equivalent.

(a) τ ∼I.

(b) If A has a cover of open sets each of whose intersection with A is I, then A is in I.

(c) If A ⊆X, then A ∧A∗= φ =⇒A ∈I.

(d) If A ⊆X, then A − A∗∈I.

(e) A ⊆ X and A is fuzzy R_I-perfect set, then A∆A∗∈I.

(f) For every fuzzy τ ∗-closed subset A, A − A∗∈I.

(g) For every A ⊆ X, if A contains no nonempty subset B with B ⊆B∗, then A ∈ I.

Proof:

To prove this theorem, it is enough to prove (d) ⇒ (e) ⇒(f). (d) ⇒(e) follows from Proposition 2.1. Suppose that A∆A∗∈I. Since A − A∗⊆ A∆A∗, by heredity property A − A∗∈I. Hence (e) ⇒ (f).

3.Fuzzy ∧∗-and fuzzy ∨∗-sets

The intent of this section is to introduce two types of sets viz. fuzzy ∧∗-sets and fuzzy ∨∗-sets

Definition 3.1. Let (X, τ, I) be an ideal topological space and A ⊆ X. We define[9]

$A^{\wedge}_{*}$= ∪{U: A ⊆ U and U is a ∗-open set}

$A^{\vee}_{*}$= ∩{F: F ⊆ A and F is a ∗-closed set}.

Some elementary but basic results concerning the above two types of sets are obtained in the following theorem:

Theorem 3.1. Let (X, τ, I) be an ideal topological space and A, B, A_α(α ∈ Δ) be subsets of X. Then the following are true:

(a) A $\subseteq A_*^\wedge$.

(b) If A is ∗-open then A = $A_*^\wedge$

(c) A ⊆ B ⇒$A_*^\wedge \subseteq B_*^\wedge$.

(d) $(A_*^\wedge)_*^\wedge = A_*^\wedge$

(e) $(\cup\{A_\alpha : \alpha \in \Delta\})_*^\wedge = \cup\{(A_\alpha)_*^\wedge : \alpha \in \Delta\}$.

(f) $(\cap\{A_\alpha : \alpha \in \Delta\})_*^\wedge \subseteq \cap\{(A_\alpha)_*^\wedge : \alpha \in \Delta\}$.

Proof.

(a) and (b) follow from the above definition.

(c) Let x∉ $B_*^\wedge$. Then there exists a ∗-open set U such that B ⊆ U and x∉U. Since A ⊆ B, x∉$A_*^\wedge$.

(d) Clearly $(A_*^\wedge)_*^\wedge \supseteq A_*^\wedge$

. By definition, $A_*^\wedge \subseteq$ U for every ∗-open set U with A ⊆ U. Then $(A_*^\wedge)_*^\wedge \subseteq U_*^\wedge$= U (by using (b) and (c)). Thus $(A_*^\wedge)_*^\wedge \subseteq A_*^\wedge$

(e) In view of (c), it is sufficient to show that $(\cup\{A_\alpha : \alpha \in \Delta\})_*^\wedge \subseteq \cup\{(A_\alpha)_*^\wedge : \alpha \in \Delta\}$.

Let x∉$\cup\{(A_\alpha)_*^\wedge : \alpha \in \Delta\}$. Then for each α ∈ Δ, there exists a ∗-open set $U_\alpha$such that$A_\alpha \subseteq U_\alpha$ and x∉U_α. Let U=$\cup\{U_\alpha : \alpha \in \Delta\}$. Then U is a ∗-open set containing $\cup\{A_\alpha : \alpha \in \Delta\}$ and x∉ U. Hence x∉$(\cup\{A_\alpha : \alpha \in \Delta\})_*^\wedge$

(f) Follows from (c) above.

Remark 3.1. In (f) of the above theorem, equality does not hold in general, even if Δ is a finite index set. We show this by the following example.

Example 3.1. Consider an ideal topological space (X, τ, I), where X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and I = {φ, {c}}. Then τ ∗ = {φ, {a}, {b}, {a, b}, {b, c}, X}. Let us consider A = {a, b} and B = {a, c}. Then = $A_*^\wedge$ {a, b}, $B_*^\wedge$= X and $(A \cap B)_*^\wedge$= {a}. Thus $(A \cap B)_*^\wedge \subsetneq A_*^\wedge \cap B_*^\wedge$.

Lemma 3.2. Let (X, τ, I) be an ideal topological space. Then $(X \backslash A)_*^\wedge$ = X \\$A_*^\vee$ for each A ⊆ X.

Proof.

We have X \ $A_*^\vee$ = X \ (∪ {F: F ⊆ A and F is a ∗-closed set}) = ∩ {X \ F: X \ A ⊆ X \ F and X \ F is a ∗-open set} = $(X \backslash A)_*^\wedge$.

Using the above lemma and Theorem 3.1, we have the following result:

Theorem 3.3. Let (X, τ, I) be an ideal topological space and A, B,$A_\alpha(\alpha \in \Delta)$ be subsets of X. Then the following are true:

(a) $A_*^\vee \subseteq$ A.

(b) If A is ∗-closed then A = $A_*^\vee$.

(c) A ⊆ B ⇒ $A_*^\vee \subseteq B_*^\vee$

(d) $(A_*^\vee)_*^\vee = A_*^\vee$

(e) $(\cap\{A_\alpha : \alpha \in \Delta\})_*^\vee = \cap\{(A_\alpha)_*^\vee : \alpha \in \Delta\}$.

(f) $\cup\{(A_\alpha)_*^\vee : \alpha \in \Delta\} \subseteq (\cup\{A_\alpha : \alpha \in \Delta\})_*^\vee$.

Definition 3.2. Let (X, τ, I) be an ideal topological space and A ⊆ X. Then A is said to be a

(i) Fuzzy $\wedge_*$-set if A = $A_*^\wedge$
(ii) Fuzzy $\vee_*$-set if A = $A_*^\vee$

Thus a subset A of X is a fuzzy $\wedge_*$-set if and only if X \ A is a fuzzy $\vee_*$-set.

Theorem 3.4. Let (X, τ, I) be an ideal topological space. Then the following statements hold:

(a) φ and X are fuzzy $\wedge_*$-sets and fuzzy $\vee_*$-sets.

(b) Every union of fuzzy $\wedge_*$-sets is a fuzzy $\vee_*$-sets.

(c) Every intersection of fuzzy $\wedge_*$-sets is a fuzzy $\vee_*$-sets.

Proof:

(a) Obvious.

(b) Let $\{A_\alpha : \alpha \in \Delta\}$ be a family of fuzzy $\wedge_*$-sets. Then $A_\alpha = (A_\alpha)_*^\wedge$, for each α ∈ Δ. Let A = $\cup\{A_\alpha : \alpha \in \Delta\}$. Then by Theorem 2.1(e),$A_*^\wedge$ = A ⇒ A is a $\wedge_*$-set.

(c) Follows from (b) above and Lemma 3.2.

4.Fuzzy *s-connected set

Theorem:4.1 If A is a fuzzy *s-connected set of an ideal space (X, τ, I) and A ⊂ B ⊂ Cl∗ (A), then B is fuzzy *s-connected. [10]

Proof:

Suppose that B is not fuzzy *s-connected. There exist fuzzy *-separated sets H and G such that B = H ∪ G. This implies that H and G are nonempty and G ∩ Cl∗ (H) = ∅ = H ∩ Cl(G). By Theorem 15, we have either A ⊂ H or A ⊂ G. Suppose that A ⊂ H. Then Cl∗ (A) ⊂ Cl∗ (H) and G ∩ Cl∗ (A) = ∅. This implies that G ⊂ B ⊂ Cl∗ (A) and G = Cl∗ (A) ∩ G = ∅. Thus, G is an empty set. Since G is nonempty, this is a contradiction. Suppose that A ⊂ G. By similar way, it follows that H is empty. This is a contradiction. Hence, B is fuzzy *s-connected.

Corollary 4.1: If A is a fuzzy *s-connected set in an ideal space (X, τ, I), then Cl∗ (A) is fuzzy *s-connected.

Theorem 4.2: If {Mi: i∈ I} is a nonempty family of fuzzy *s-connected sets of an ideal space (X, τ, I) with $\bigcup_{i\in I} M_i \neq \emptyset$, then $\bigcup_{i\in I} M_i$ is fuzzy *s-connected.

Proof.

Suppose that $\bigcup_{i\in I} M_i$ is not fuzzy *s -connected. Then we have $\bigcup_{i\in I} M_i$ = H ∪ G, where H and G are fuzzy *-separated sets in X. Since $\bigcup_{i\in I} M_i \neq \emptyset$, we have a point x in $\bigcup_{i\in I} M_i$. Since x ∈$\bigcup_{i\in I} M_i$, either x ∈ H or x ∈ G. Suppose that x ∈ H. Since x ∈ $M_i$for each i ∈ I, then M_i and H intersect for each i ∈ I. By Theorem 14, M_i⊂ H or Mi ⊂ G. Since H and G are disjoint, M_i⊂ H for all i∈ I and hence $\bigcup_{i\in I} M_i$ ⊂ H. This implies that G is empty. This is a contradiction. Suppose that x ∈ G. By similar way, we have that H is empty. This is a contradiction. Thus, $\bigcup_{i\in I} M_i$ is fuzzy *s -connected.

Definition 4.1: Let X be an ideal space and x ∈ X. The union of all fuzzy *s -connected subsets of X containing x is called the fuzzy *-component of X containing x.

Theorem 4.2. Each fuzzy *-component of an ideal space (X, τ, I) is a maximal fuzzy *s -connected set of X.

Theorem 4.3. The set of all distinct fuzzy*-components of an ideal space (X, τ, I) forms a partition of X.

Proof.

Let A and B be two distinct fuzzy*-components of X. Suppose that A and B intersect. Then, A ∪ B is fuzzy *s -connected in X. Since A ⊂ A ∪ B, then A is not maximal. Thus, A and B are disjoint.

Theorem 4.4. Each fuzzy *-component of an ideal space (X, τ, I) is *-closed in X.

Proof.

Let A be a fuzzy *-component of X. By Corollary 16, Cl∗ (A) is fuzzy *s- connected and A = Cl∗ (A). Thus, A is *-closed in X.

5.Fuzzy R_I-Topology

By Corollary 2.1 and Proposition 2.11, we observe that the collection R satisfies the conditions of being a basis for some fuzzy topology and it will be called as R_I ∗(τ,I). We define

R_I(τ,I) = {A⊆X/X−A∈R_I∗(τ,I)} on an on-empty set X. Clearly, R_I (τ, I) is a fuzzy topology if the set X is finite. The members of the collection R_I(τ,I) will be called fuzzy R_I-open sets. If there is no confusion about the fuzzy topology τ and the ideal I, then we call R_I(τ, I) as R_I - fuzzy topology when X is finite.

Definition 5.1. A subset A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy R_I closed if it is a complement of a fuzzy R_I-open set.

Definition 5.2. Let A be a subset of a fuzzy ideal topological space (X, τ, I). One defines R_I - interior of the set A as the largest fuzzy R_I -open set contained in A. One will denote R_I -interior of a set A by R_I − int(A).

Definition 5.3. Let A be a subset of a fuzzy ideal topological space (X, τ, I). A point x ∈ A is said to be an R_I -interior point of the set A if there exists an fuzzy R_I-open set U of x such that x ∈ U ⊆A.

Definition5.4. Let (X,τ,I)be a fuzzy ideal topological space and x∈X. One defines $R_I$neighbourhood of x as a fuzzy R_I-open set containing x. One denotes the set of all $R_I$neighbourhoods of x by R_I −N (x).

Proposition 5.1. In a fuzzy ideal topological space (X, τ, I), every fuzzy τ ∗-open set is a fuzzy $R_I$open set.

Proof:

Let A be a fuzzy τ ∗-open set. Therefore, X − A is a fuzzy τ ∗-closed set. That implies that X − A is a fuzzy R_I -closed set. Hence A is a fuzzy R_I -open set.

Corollary 5.1. The fuzzy topology R_I (τ, I) on a finite set X is finer than the fuzzy topology τ∗ (τ, I).

Proof :

The proof follows from Proposition 5.1.

Corollary 5.2. For any subset A of a fuzzy ideal topological space (X, τ, I), int(A) is an fuzzy R_I open set.

Proof :

The proof follows from Proposition 5.1.

Remark 5.1.

(a) Since every fuzzy open set is a fuzzy R_I -open set, every neighbourhood U of a point x ∈ X is an fuzzy R_I -neighbourhood of x.

(b) If x ∈ X is an interior point of a subset A of X, then x is an fuzzy R_I -interior point of A.

(c) From (b), we have int(A) ⊆ int∗(A) ⊆R_I − int(A), where int∗(A) denotes interior of A with respect to the fuzzy topology τ ∗.

Theorem 5.1. Let A and B be subsets of a fuzzy ideal topological space (X, τ, I) with X being finite. Then the following properties hold.

(a) R_I − int(A) = ∨ {U: U ⊆ A and U is an fuzzy R_I-open set}.

(b) R_I− int(A) is the largest fuzzy R_I-open set of X contained in A.

(c) A is fuzzy R_I − open if and only if A = R_I −int(A).

(d) R_I−int∗(R_I−int(A))=R_I−int(A). (e) If A ⊆B, then R_I − int(A) ⊆R_I −int(B).

Proof :

The proof follows from Definitions 5.2,5.3,5.4.

Definition5.5. Let A be subset of a fuzzy ideal topological space (X, τ, I). One defines fuzzy R_I closure of the set A as the smallest R_I -closed set containing A. One will denote fuzzy R_I -closure of a set A by R_I −cl(A).

Remark 5.2. For any subset A of a fuzzy ideal topological space (X, τ, I), R_I−cl(A) ⊆ cl∗(A) ⊆ cl(A).

Theorem 5.2. Let A and B be subsets of a fuzzy ideal topological space (X, τ, I) where X is finite. Then the following properties hold:

(a) R_I − cl(A) = ∧ {F: A ⊆ F and F is fuzzy R_I -closed set}.

(b) A is fuzzy R_I-closed if and only if A = R_I − cl(A).

(c) R_I−cl(R_I−cl(A)) = R_I−cl(A)

(d) If A ⊆B, then R_I − cl(A) ⊆R_I −cl(B).

Proof :

The proof follows from Definition 5.5.

Theorem 5.3. Let A be subsets of a fuzzy ideal topological space (X, τ, I). Then the following properties hold:

(a) R_I−int(X−A) = X−R_I−cl(A);

(b) R_I−cl(X−A) = X−R_I−int(A).

Proof:

The proof follows from Definitions 5.1,5.2,5.5.

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