

GEOGRAPHIC INFORMATION STUDY ON SPATIAL OBJECTS USING FUZZY TOPOLOGY

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Abstract

Fuzzy topology can help us to deal with the spatial objects, which have values with Vagueness. In this paper, Fuzzy topology has been used to model the spatial relation between the spatial objects. In calculating the spatial relation between the objects, concepts of fuzzy topology had been employed to calculate the relation between the fuzzy objects. We knew that air pollution is a great threat to our environment in recent areas. In order to calculate the pollution level of an particular place we consider spatial relation between two 3-D fuzzy object like moving air pollution cloud and density of population. The relation between them is calculated as Fuzzy intersection matrix. Similarity of the matrix is used to calculate the pollution level of a place.

Introduction

Geographic Information System (**GIS**) is a digitalised way of storing the geographical data based on the location of the Earth's surface. It can help an individual or an organisation to understand the spatial pattern of a particular place for the better benefits. The GIS data along with the geographical data provides the data about people. It can help us to improve the allocation of the resources with the better planning. It can help us to manage the expenses by allocating resources properly.

Human life depends more on the environmental conditions. Air is very vital for every human to live, but the quality of air is becoming worse. Air pollution is caused by solid, liquid particles and certain gases that arises from the vehicles mostly. Long term exposure to air pollution can cause many diseases that are associated with heart and lungs, cancer and other health problems.

Nowadays, the pollution level of Air is being increased and it makes the air very worse for the humans to breathe. The quality of air has been monitored for the precaution level, so that prevention measures can be taken in advance.

Fuzzy properties of Spatial Objects

In order to calculate the pollution level, we implement the fuzzy logic idea here. As it can be used to calculate the vagueness. In crisp values, we can tell whether the fan is on or off i.e., but using fuzzy values we can also tell about the fan speed. We have used the concepts of fuzzy in modelling the relation between the cloud and population density.

Data abstraction and digitalization in GIS environment by defining spatial dimension, boundary and different attributes for each object in real world. It is necessary to interpret these data as mathematically in order to improve the accuracy and efficiency.

By set theory we can define the region as (C) is consisted of interior (C°), boundary (∂C), exterior (C_e). 9-intersection matrix Egen Hofer matrix uses the concepts of crisp values.

Definition 1:

For 3D fuzzy region C , its membership function can be defined as

$$C = \{(x, y, z), \mu_C(x, y, z)\} \text{ where } (x, y, z) \in R^3 \text{ and } \mu_C: R^3 \rightarrow [0,1]$$

$\mu_C(x, y, z)$ is the membership function for the 3D region.

Definition 2:

A simple region is known as C , its interior is defined as follows.

$$\mu_{C^\circ}(x, y, z) = \begin{cases} \mu_C(x, y, z) & \text{for } (x, y, z) \in \text{supp}(C^\circ) \\ 0 & \text{otherwise} \end{cases}$$

Definition 3:

If C is a fuzzy region the boundary of C is defined as follows.

$$\mu_{\partial C}(x, y, z) = 2 \min [\mu_C(x, y, z), 1 - \mu_C(x, y, z)]$$

Definition 4:

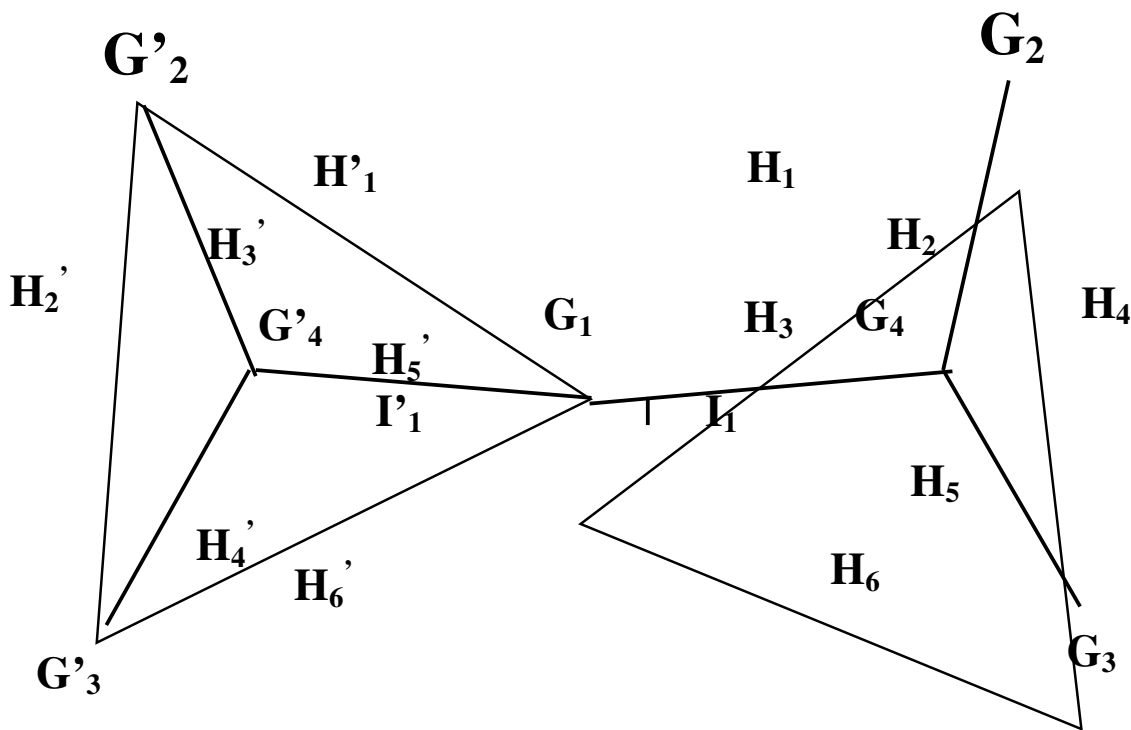
If C is a fuzzy region their exterior is defined as follows

$$\mu_{C_e}(x, y, z) = \begin{cases} \mu_C(x, y, z) & \text{for } (x, y, z) \in \text{supp}(C_e) \\ 0 & \text{otherwise} \end{cases}$$

Topological Relation

Egen hofer 9- intersection matrix is implemented to interpret the 3-D topological relations between the objects. Here, we divide the object into Points (G), Lines(H) Surface and body. Based on the topological dimension there can be many spatial relation between these two 3-D objects. There can be many kind of relation between these two objects. We have four topological elements to calculate the relation between them. Binary topological relation between two objects can be calculated according to the intersection of C's interior, exterior and boundary with the intersection of D's interior, exterior and boundary.

Topological Relation between two simple objects



Elements in C can be related to the elements of D. After that we can tell the number of relation between the objects based on their dimension.

No. of topological relation between two bodies

512*	ODD	1DD	2DD	3DD
ODC	16	24	16	4
1DC	24	36	24	6
2DC	16	24	16	4
3DC	4	4	4	1

By considering the values 0 (empty) and 1 (non-empty) we can provide $2^9 = 512$ relations between them. But in the table we have 512 relations but these relations cannot be found in our life and we cannot extract these relationships between these objects, only eight relations can be interpreted and studied in real life. The relation is studied between the cloud and population density as 3-D objects. The values of these 8 matrixes are calculated using the idea of Egen Hofer 9- intersection matrix with the intersection of the interior, exterior, boundary of C and with the intersection of the interior, boundary and exterior of D.

We can create 9-intersection matrix for the eight topological relation.

$$U(C, D) = \begin{pmatrix} C^\circ \cap D^\circ & C^\circ \cap \partial D & C^\circ \cap D_e \\ \partial C \cap D^\circ & \partial C \cap \partial D & \partial C \cap D_e \\ C_e \cap D^\circ & C_e \cap \partial D & C_e \cap D_e \end{pmatrix}$$

$$\chi_r(u) = \begin{cases} 0 & u = 0 \\ 1 & \text{otherwise} \end{cases}$$

χ_r - is the characteristic function that can be used to know whether the intersection is empty or nonempty.

Fuzzy nine-intersection matrix

Fuzzy nine matrix is an generalisation concept from the crisp nine-intersection matrix. If fuzzy nine intersection matrix between C and D is shown by F_9 then

$$\mu_{F_9}(u) = H(u) \text{ for all } u \in U$$

where H is “height” or the maximum of the intersectionmembership value for C and D. Then, the result will be thefollowing notation.

$$F(C, D) = \begin{pmatrix} H(C^\circ \cap D^\circ) & H(C^\circ \cap \partial D) & H(C^\circ \cap D_e) \\ H(\partial C \cap D^\circ) & H(\partial C \cap \partial D) & H(\partial C \cap D_e) \\ H(C_e \cap D^\circ) & H(C_e \cap \partial D) & H(C_e \cap D_e) \end{pmatrix}$$

If we are studying any two objects that has crisp or fuzzy topological relations. In order to study about the similarity between the objects, we have to study about the type and the strength of the relation between the objects.

With the 9- intersection matrix, interior, exterior and boundary only can be studied. With the 4x4 intersection matrix, we can study the interior, exterior, boundary and boundary of the boundary.

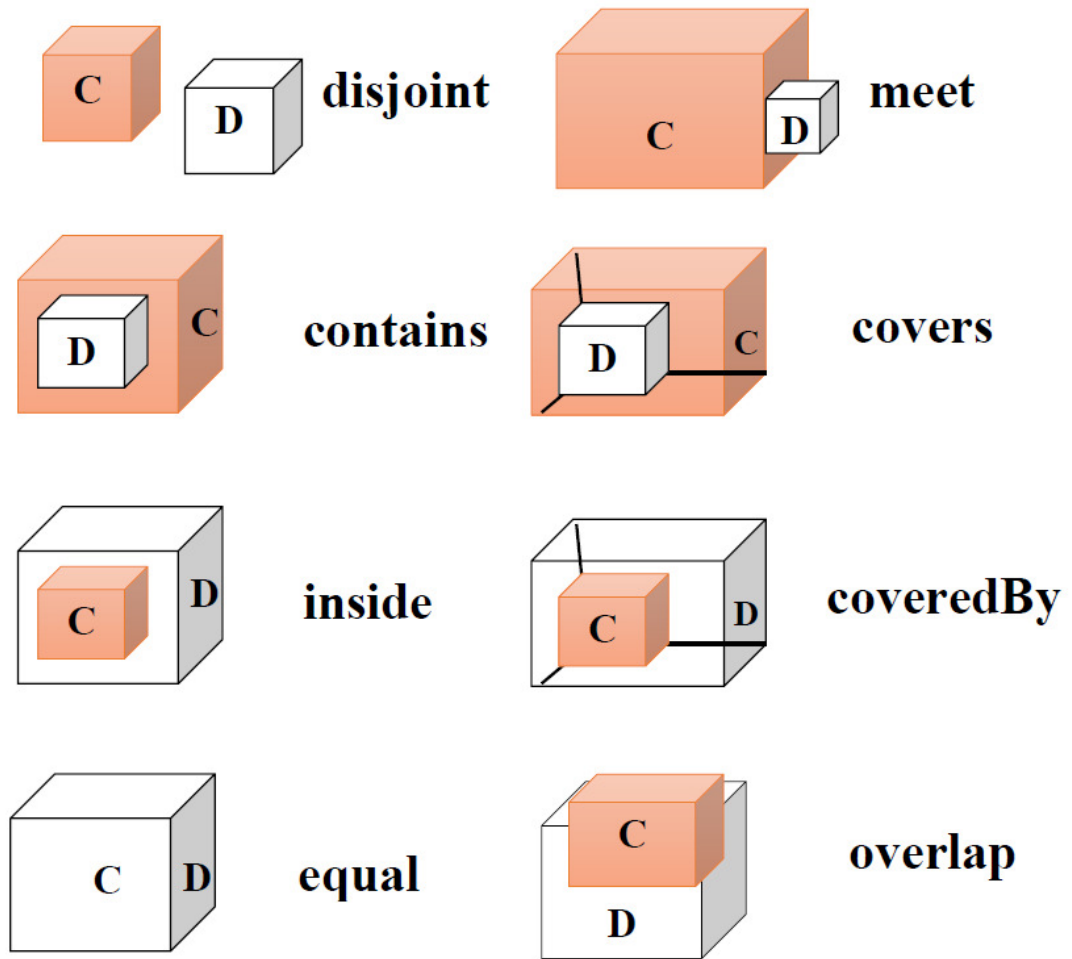
$$R(C_\alpha D_\beta) = \begin{pmatrix} \partial C_\alpha^- \cap \partial D_\beta^- & \partial C_\alpha^- \cap \partial D_\beta & \partial C_\alpha^- \cap \partial D_\beta^\circ & \partial C_\alpha^- \cap D_\beta^\circ \\ \partial C_\alpha \cap \partial D_\beta^- & \partial C_\alpha \cap \partial D_\beta & \partial C_\alpha \cap \partial D_\beta^\circ & \partial C_\alpha \cap D_\beta^\circ \\ \partial C_\alpha^\circ \cap \partial D_\beta^- & \partial C_\alpha^\circ \cap \partial D_\beta & \partial C_\alpha^\circ \cap \partial D_\beta^\circ & \partial C_\alpha^\circ \cap D_\beta^\circ \\ C_\alpha^\circ \cap \partial D_\beta^- & C_\alpha^\circ \cap \partial D_\beta & C_\alpha^\circ \cap \partial D_\beta^\circ & C_\alpha^\circ \cap D_\beta^\circ \end{pmatrix}$$

∂C_α^- – is the outer boundary of the region

∂C_α° – is the inner boundary of the region

$C_\alpha = \partial C_\alpha^- - \partial C_\alpha^\circ$ – is the interior of the region.

Possible topological relation between two objects



This matrix is the result of all the possible relation between two objects in a fuzzy region. We can create many relations between these two objects. Almost,65,536 relations can be defined. Here, we consider only the possible 8 relations that can be interpreted in real life.

Relation	Matrix
Disjoint	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$
Meet	$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

Contains	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$
Covers	$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$
Inside	$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$
Covered By	$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$
Over lap	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Equal	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$

The fuzzy values of this matrix had to be found for the process of finding similarity between the matrixes.

$$\mu_{F_A}(u) = H(u) \text{ for all } u \in U$$

where H is “height” or the maximum of the intersection membership value for C and D. Then, the result will be the following notation.

$$F(C_\alpha D_\beta) = \begin{pmatrix} H(\partial C_\alpha^- \cap \partial D_\beta^-) & H(\partial C_\alpha^- \cap \partial D_\beta) & H(\partial C_\alpha^- \cap \partial D_\beta^\circ) & H(\partial C_\alpha^- \cap D_\beta^\circ) \\ H(\partial C_\alpha \cap \partial D_\beta^-) & H(\partial C_\alpha \cap \partial D_\beta) & H(\partial C_\alpha \cap \partial D_\beta^\circ) & H(\partial C_\alpha \cap D_\beta^\circ) \\ H(\partial C_\alpha^\circ \cap \partial D_\beta^-) & H(\partial C_\alpha^\circ \cap \partial D_\beta) & H(\partial C_\alpha^\circ \cap \partial D_\beta^\circ) & H(\partial C_\alpha^\circ \cap D_\beta^\circ) \\ H(C_\alpha^\circ \cap \partial D_\beta^-) & H(C_\alpha^\circ \cap \partial D_\beta) & H(C_\alpha^\circ \cap \partial D_\beta^\circ) & H(C_\alpha^\circ \cap D_\beta^\circ) \end{pmatrix}$$

The crisp relations can be associated with a characteristic function $\chi_r(v)$, where $r \in R_8$ possesses the properties of fuzzy sets. Comparison between F_4 and R_8 can be formalized as a fuzzy relation $\emptyset(F_4, r)$ for all $r \in R_8$. Then, the membership function $\mu_\emptyset(F_4, r)$ can represent the similarity between F_4 and r .

$$\mu_\emptyset(F_4, r) = \gamma[(F_4 \wedge r) \vee (F_4^- \wedge r^-)]$$

Here,

1. \wedge represents the minimum value of fuzzy.
2. \vee represents the maximum value of fuzzy.
3. F_4^- represents the complement of F_4 .
4. r^- represent the complement of r .
5. γ - minimum of membership.

$\mu_\emptyset(F_4, r)$ is used to calculate the amount of similarity between F_4 and $r_1, r_2, r_3, r_4, r_5, r_6, r_7$ and r_8 . $\mu_\emptyset(F_4, r)$ is used to determine the strength of 3D topological relations.

Decision Variables

With the calculation of resemblances between matrixes quad set characterized as:

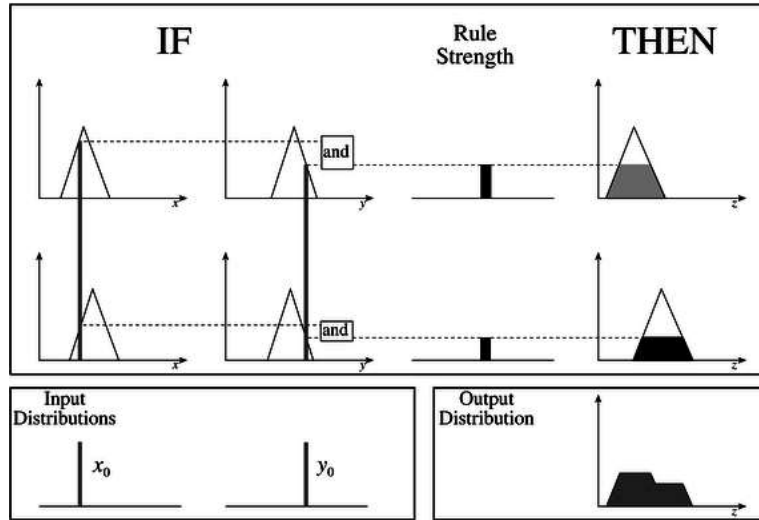
$$Q = \{q(r_i), n(r_i), q(r_j), n(r_j)\}$$

In which, $n(r_i)$ and $n(r_j)$ are the two closest relations and are called the superior and the sub-superior relations, and $q(r_i)$ and $q(r_j)$ are their quantifiers. If the quantifier of the superior relation r_j is “no”, only the sub-superior relation is applied.

Classifying and labelling each class with a natural language term, the strength of relations will be defined using “no”, “slightly”, “somewhat”, “mostly”, and “clearly” terms.

$$q(r) = \begin{cases} \text{'clearly'} & \text{if } 0.9 < \mu_\emptyset(F_4, r) \\ \text{'mostly'} & \text{if } 0.7 < \mu_\emptyset(F_4, r) \leq 0.9 \\ \text{'somewhat'} & \text{if } 0.5 < \mu_\emptyset(F_4, r) \leq 0.7 \\ \text{'slightly'} & \text{if } 0.02 < \mu_\emptyset(F_4, r) \leq 0.5 \\ \text{'no'} & \text{if } \mu_\emptyset(F_4, r) \leq 0.02 \end{cases}$$

A spatial knowledgebasedsystemis designed for alarming in a control centre usingdecision variables and fuzzy “If–Then” rules.



The centre is toanalyse and decide on 3D fuzzy topological relations betweenpollution cloud and a residential area, using a spatialknowledge-based system.The system will help us to know the quality of the air and it can be used to warn us. The system does not allow the decision variable to remain as a constant one, it may change with respect to the quality of the air.

With the help of the topology, we can understand the quality of air it allows us to understand the relation between the population density and pollution cloud. If-Then rules are implemented to make the system take their decision. In both control theory and the theoryof approximate reasoning, much of the knowledge aboutsystem behaviour and system control can be stated in the formof If–Then rules.

Relation	$q(r)$
Disjoint	0.00
Meet	0.07
Contains	0.28
Covers	0.16
Inside	1
Covered by	0.68
Equal	0.89
Overlap	0.83

Conclusion

Air pollution is a growing problem in every development nation. Many countries have started to take serious steps to control them. The first step in preventing pollution is to predict pollution level.

Major work of the article is to study the spatial relation between the two objects that are three dimensions, we have employed fuzzy nine intersection matrix to calculate the fuzzy values for all the possible real-life situations. The Pollution cloud and Population density are considered as the two 3-D objects. By identifying the intersection matrix for the relation that can be interpreted in our real-life is considered.

Then the Strength of the relation ($q(r)$) value has been calculated, by which we can understand the relation between two three-dimension objects. This can also help us to understand the intensity of the relation between these objects. If their relation is stronger, we can come into a conclusion that, the pollution level is higher. This can be really helpful in real world situation. Because after knowing the pollution level of a particular place, government can take certain measure to bring down the pollution level. This will be helpful for the present and future generations.

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