HIGHLY TOTALLY REGULAR AND NEIBOURLYTOTALLY REGULAR FUZZY GRAPH

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ABSTRACT

In this paper ,highly totally regular fuzzy graph is introduced.A necessary and sufficient condition under which highly regular and highly totally regular fuzzy graphs are equivalent isprovided. A comparative study between highly regular, highlytotally regular and neighbourly totally regular fuzzy graphs ismade. Some results on highly totally regular fuzzy graphs areestablished.

Keywords: Degree of vertex in fuzzy graph, total degree of fuzzy graph, neighbourly totally regular fuzzy graph, highly regular fuzzy graph.

1. INTRODUCTION

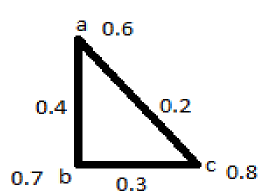
Fuzzy Graph theory was introduced by AzrielRozenfeld in 1975. Though it isvery young it has been growing very fast and has numerous application in various fields. A.NagoorGani and S.R. Latha introduced neighbourly regular and highly regular fuzzy graphs. That paper motivate us to introduce highly totally regular fuzzy graph and give some results using highly regular and highly totally regular fuzzy graph. The necessary and sufficient condition for highly regular fuzzy graph to be highly totally regular fuzzy graph and vice versa is established. Through out this paper two main notations are used, σ whichis the fuzzy vertex function and μ which is fuzzy edge function.

2. PRELIMINARIES

Definition 2.1

A fuzzy graph is a pair of functions G: (σ, μ) where σ : V→[0,1] is a fuzzy subset of non-empty set V and μ : V xV→[0,1] is symmetric fuzzy relation on σ such that for all x,y in V the condition μ(u,v) ≤ σ(u) Λ σ(v) is satisfied for all (u,v) in E.

Example:

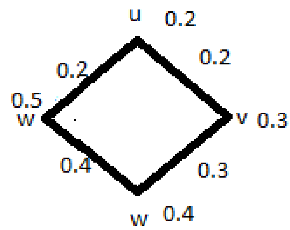


FuzzyGraph (fig 2.1)

Definition 2.2

A fuzzy graph G is said to be complete fuzzy graph if μ(u,v) = σ(u) Λ σ(v) is satisfied for all (u,v) in E.

Example:

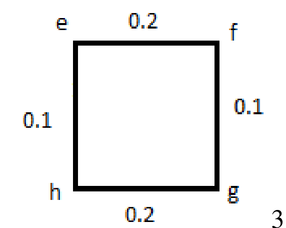


complete fuzzy graph (fig 2.2)

Definition 2.3

A fuzzy graph G is said to be regular fuzzy graph,if each vertex has same degree .

Example:

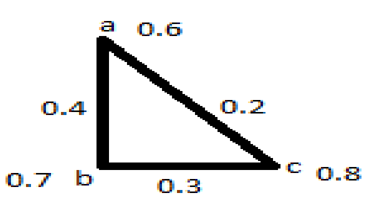


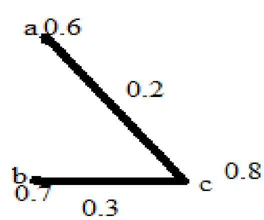
Regular fuzzy graph (Fig 2.3)

Definition 2.4

Let G : (σ, μ) be fuzzy graph and let o, p be two same vertices. An edge o p is called fuzzy bridge if deletion of o p reduces the strength of connectedness between the pair of vertices.

Example:



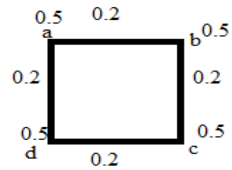


Fuzzy bridgeThe strength of connectedness

Definition 2.5

Let G : (σ, μ) be fuzzy graph such that G*:(V,E) is a cycle. Then G is fuzzy cycle if and only if there does not exists unique edge (x,y) such that μ(x,y) = Λ { μ(u,v) /(u,v) Є E}

Example



Fuzzy cycle

A Graph G is does not exist unique edge .

Definition 2.6

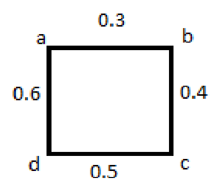
A fuzzy graph H:(P, υ, τ) is calledfuzzy subgraph G:(V, σ, μ) induced by P if P⊆V,v(x)⊆ σ(x) and τ(x) ⊆ μ(x).

Definition 2.7

The degree of vertex is defined as the sum of weight of edges incident with a vertex. It is denoted by d(u).

Example:

Degree of vertex (a) = d(a) =μ(a, b)+μ(a, d)

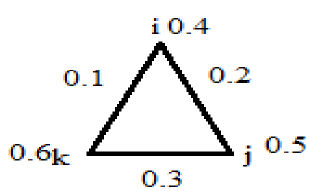


Definition 2.8

Let G : (σ, μ) be fuzzy graph on G*:(V,E). The total degree of a vertex is defined as td(u) = ∑ μ (u,v) + σ(u) = d(u) + σ(u).

Example:

td(j) =d(j)+ σ(j)=1.0

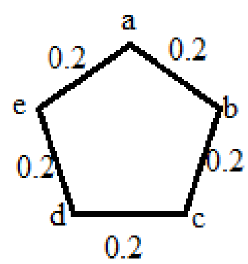


Total degree of vertex(fig 2.8)

Definition 2.9

A G is said to be neighbourly regular fuzzy graph if every two adjacent vertices of G has samedegree.

Example: d(a)=d(b); d(b)=d(c); d(c)=d(d); d(d)=d(e); d(a)=d(e).

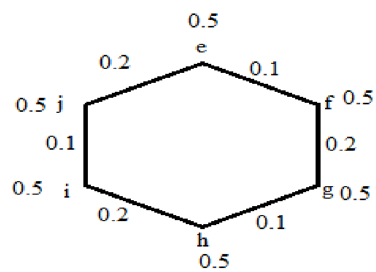


Neighbourly fuzzy graph (fig 2.9)

Definition 2.10

G is said to be neighbourly totally regular fuzzy graph if every two adjacent vertices has same total degrees.

Example:



td(e)= td(f) ; td(f) = td(g) ; td(g) = td(h); td(h) = td(i); td(i) =td(j); td(e) = td(j).

Neighbourly totally fuzzy graph (Fig 2. 10)

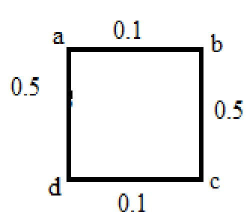
3. HIGHLY TOTALLY REGULAR FUZZY GRAPH

Definition 3.1

A graph G is said to be highly regular fuzzy graph if every vertex of G is adjacent to vertices with same degrees.

Example:

d(a)=d(b)=d(c)=d(d)



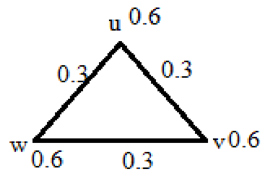
Highly regular fuzzy graph (Fig 3.1)

Definition 3.2

A graph G is said to be highly totally regular fuzzy graph if every vertex of G is adjacent to vertices with same total degree.

Example:

td(u) =td(v)= td(w)

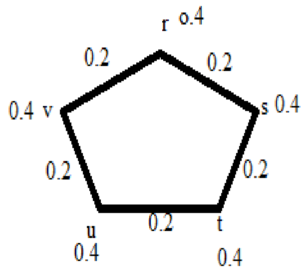


Highly totally regular fuzzy graph(Fig 3.2)

Proposition 3.3

A highly totally regular fuzzy graph is alsoneighbourly totally regular fuzzy graph.

Example 3.4



(fig 3.3)

Consider a G:(σ, μ) be fuzzy graph. Define σ(r) = 0.4, σ(s) = 0.4, σ(t) = 0.4, σ(u) = 0.4, σ(v) = 0.4, μ(r,s)= 0.2, μ(s,t)=0.2, μ(t,u)=0.2, μ(u,v)=0.2, μ(r,v) =0.2.

A graph G is highly totally regular fuzzy graph.

td(r) = d(r)+ σ(r)= 0.8

td(s) = td(t) = td(u) = td(v) = 0.8

If every vertex of G is adjacent to vertices with same degree .

A graph G is neighbourly totally regular fuzzy graph.

td(r) = td(s) = d(r)+σ(r)=d(s)+ σ(s)

td(r) = td(s) = 0.8

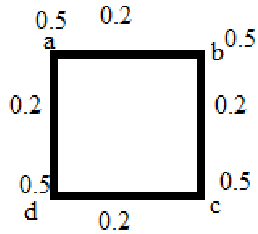
td(s) = td(t); td(t) = td(u); td(u) = td(v); td(r) = td(v) =0.8

If every two vertex of G is adjacent to vertices with same degree.

G is highly totally regular fuzzy graph then G is also neighbourly totally regular since every two adjacent vertices has the same total degree.

Proposition 3.5

A highly totally regular fuzzy graph is also highly regular fuzzy graph.



Example

Consider a G:(σ, μ) be fuzzy graph. Define σ(a) = 0.5, σ(b) = 0.5, σ(c) =0.5, σ(d) =0.5,μ(a,b)=0.2, μ(b ,c)=0.2, μ(c, d)=0.2, μ(a ,d)=0.2,

A graph G is highly totally regular fuzzy graph.

td(x) = td(y) = td(z) = 0.9

If every vertex of G is adjacent to vertices has same total degree.

A graph G is highly regular fuzzy graph.

d(x) = d(y) = d(z) =0.7

If every vertex of G is adjacent to vertices has same degree.

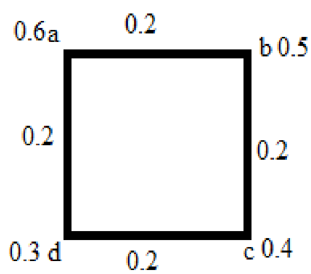
A graph is highly totally regular fuzzy graph then G is also highly regular fuzzy graph .since every adjacent vertices with same degree.

Proposition 3.7

A highly regular fuzzy graph is need not be highly totally regular fuzzy graph.

Example 3.8

Fig 3.8



Consider a G:(σ, μ) be fuzzy graph. Define σ(a) =0.6, σ(b) = 0.5, σ(d) =0.4, σ(h) = 0.3,μ(a,b)= 0.2, μ(b, c)=0.2, μ(c, d)=0.2, μ(a, d)=0.2.

A graph G is highly fuzzy graph.

d(e) = d(f) = d(g) = d(h) = 0.4

If every vertex G is adjacent to vertices has same degree.

A graph G is not highly totally regular fuzzy graph.

td(a) ≠ td(b) ≠ td(c) ≠ td(d)) ; 1.0≠ 0.9 ≠ 0.8≠ 0.7

A graph is highly regular fuzzy graph but G is need not be highly totally regular fuzzy graph .since every vertex has same degree .hence G is highly regular fuzzy graph

Proposition 3.9

A neighbourly totally regular fuzzy graph is also highly totally regular fuzzy graph.

Example 3.10

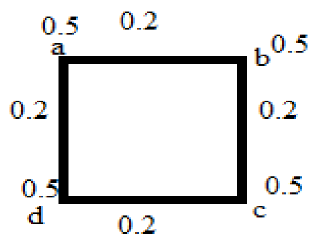


Fig 3.10

Consider a G:(σ, μ) be fuzzy graph. Define σ(a) = 0.5, σ(b) = 0.5, σ(c) = 0.5, σ(d) = 0.5,

μ(a, b)= 0.2, μ(b ,c)=0.2, μ(c ,d)=0.0.2, μ(a ,d)=0.2

A graph G is neighbourly totally fuzzy graph.

td(a) = td(b); td(b) = td(c); td(c) = td(d); td(a) = td(d) = 1.0

If every two vertex of G is adjacent vertices has same total degree.

A Graph G is highly totally regular fuzzy graph.

td(a) =td(b) = td(c)=td(d) = 1.0

If everyvertex of G is adjacent vertices has same total degree.

G is neighbourly totally regular fuzzy graph and also highly totally regular since every adjacent vertices has the same total degree.

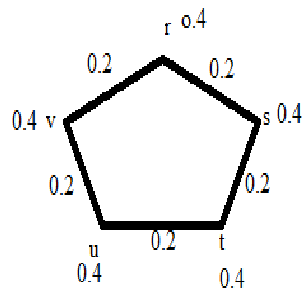
Theorem 3.11

Let G:(σ, μ) is need not be complete fuzzy graph such that G*:(V,E) is not cycle then G is both highly regular and highly totally regular fuzzy graph if and only if the degrees of all the vertices are same.

Proof:

Let G is need not be complete fuzzy graph with vertices $u1, u2, \ldots \ldots . un$ such that G is highly regular and highly totally regular fuzzy graph. Let the adjacent vertices of $u1$ be $u2, u3, \ldots \ldots . un$ with degrees $k2, k3, \ldots . kn$ Since G is highly regular $k2, k3, \ldots . kn$. Assume Suppose d($u1$)≠d($u2$), let vk be the vertex adjacent to u3. If σ not is a constant function thentd(vk))≠td($u1$).So $u3$ is adjacent to vk and $u1$ with distinct total degree→G is not highlytotally regular. Since G is complete fuzzygraph σ can notbe constant function. Sod($u1$) can has degree one of. Hence the degrees $k2, k3, \ldots . . kn$of all the vertices are not same. Conversely the degrees of all the vertices are same. This means that each vertex is adjacent to vertices with same degrees. Hence G is highly regular. Since G is not complete fuzzy graph, the total degrees of all the vertices are same. Hence G is highly totally regular.

Example 3.12



Highly regular and Highly totally regular fuzzy graph (Fig 3.12)

Consider a G:(σ, μ) be fuzzy graph. Define σ(r) = 0.4, σ(s) = 0.4, σ(t) = 0.4, σ(u) = 0.4, σ(v) = 0.4, , μ(r, s) = 0.2, μ(s, t)=0.2, μ(t, u)=0.2, μ(u, v)=0.2, μ(r, v)=0.2. A graph G is not complete fuzzy graph . μ(a, b) = σ(a) Λσ(b) then G is highly regular and highly totally regular fuzzy graph.

4. PROPERTIES OF HIGHLY TOTALLYREGULAR FUZZY GRAPH

Theorem 4.1

Let G:(σ, μ) be fuzzy graph. If G is highly regular fuzzy graph and σ isconstant function then G is highly totallyregular fuzzy graph.

Proof:

Assume G is highly regular fuzzygraph. i.e; every vertex is adjacent tovertices with same degrees. Let u be thevertex adjacent to u1 and u2 with samedegrees $k1$ and $k2$→d(u1)=$k1$, d(u2)=$k2$where$k1 = k2$. Also assume σ(u)=c for all u ЄV.

ToProve:

td(u1) =td(u2). Suppose td(u1)≠td(u2)→ $k1 + c \neq k2 + c$→$k1 \neq k2$. Which is contradiction td(u1) =td(u2).Hence u is adjacentto vertices with same total degrees. HenceG is highly totally regular fuzzy graph.

Theorem 4.2

Let G:(σ, μ) be fuzzy graph. If G is need not be highly totally regular fuzzy graph and σ is not a constant function then G is highly regular fuzzy graph.

Proof.

Assume G is highly totally regular fuzzy graph. (i.e): u is adjacent to vertices u1 and u2 with same total degrees. Let d(u1)= $k1$ and d(u2)=$k2$. Also assume σ(u)=c for all u ЄV .Since td(u1) =td(u2), σ(u1)+d(u1) =σ(u2)+d(u2)→$c +$

$k1 = c + k2 \rightarrow k1 = k2$. So u isadjacent to vertices u1 and u2 with samedegrees. Which is contradiction toσ is not constant function then u is adjcent to vertex with not same total degrees.but every vertex has same degree.This is true for each vertex in G.So G is highly regular.

Proposition 4.3

Let G:(σ, μ) be fuzzy graph. If G is both highly regular and highly totally regular then σ is aconstant function.

Example 4.4

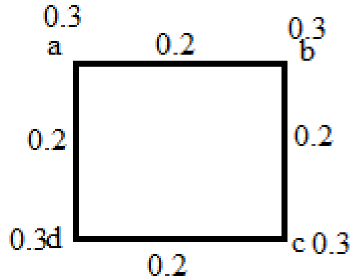


fig4.3

Consider a G:(σ, μ) be fuzzy graph. Define σ(a) = 0.3, σ(b) = 0.3, σ(c) = 0.3, σ(d) = 0.3, μ(a ,b) = 0.2, μ(v,w)=0.1, μ (b, c)=0.2, μ(a ,d) = 0.1.

A graph G is highly regular fuzzy graph and also highly totally regular fuzzy graph.

d(a) = d(b) =d(c) = d(d) =0.4

If every adjacent vertex has same degree. and

td(a) =td(b) = td(c) =td(d) = 0.7

If every adjacent vertex has same total degree.

The graph is highly regular and highly totally regular then σ is constant function.

Proposition 4.5

Let G be either highly regular or highly totally regular fuzzygraph where G* :(V ,E) is cycle. Then G is need not be fuzzy cycle.

Example 4.6

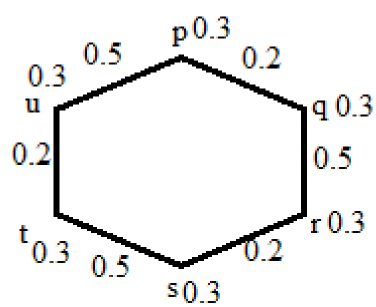


Fig4.6

Consider a G:(σ, μ) be fuzzy graph. Define σ(c) = 0.3, σ(d) = 0.3, σ(e) = 0.3, σ(f) = 0.3, μ(c,d)= 0.2, μ(d, e)=0.5, μ (e, f)=0.2, μ(c, f)=0.5.

A graph G is highly regular fuzzy graph.

d(p) = d(q) = d(r) = d(s) = d(t) = d(u) =0.7

If every adjacent vertex has same degree.

A graph G is highly totally fuzzy graph.

td(p) = td(q) = td(r) = td(s) = td(t) = td(u) = 1.0

If every adjacent vertex has same total degree.

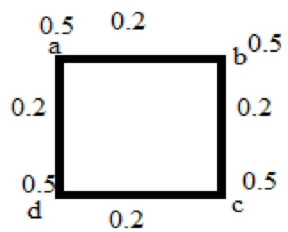
The graph is highly regular and highly totally regular but G is does not exist unique edge.Hence G is not fuzzy cycle.

Proposition 4.7

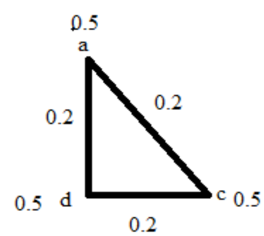
Let G be highly regular fuzzy graph. Then the fuzzy subgraph of G is also highly regular fuzzy graph.

Example 4.8

Highly regular fuzzy graph



Fuzzy sungraph



Consider a G:(σ, μ) be fuzzy graph. Define σ(a) = 0.3, σ(b) = 0.3, σ(c) = 0.3, σ(d) = 0.3, μ(a,b)= 0.2, μ(b,c)=0.2, μ (c,d)=0.2, μ(a,d)=0.2. Consider a H1:(υ, τ) be fuzzy graph. Define υ(x) =0.2, υ(y) = 0.2, τ(x ,y)= 0.5, τ(y, z) =0.5, τ(z, x)=0.5. Here G is highly regular fuzzy graph and the fuzzy subgraph in G is also every adjcent vertex has same degree. Hence the fuzzy subgraph in G is highly regular fuzzy graph.

Proposition 4.9

If G is complete fuzzy graph then Gc need not highly totally regular fuzzy graph.

Example 4.10

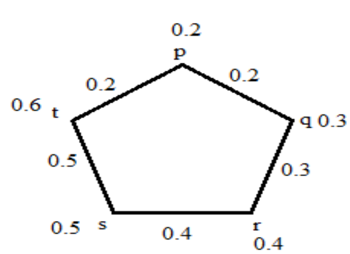


Fig 4.10

Consider a G:(σ, μ) be fuzzy graph. Define σ(a) = 0.2, σ(b) = 0.3, σ(c) = 0.4, σ(d) = 0.5, σ(e) = 0.6, μ(a,b)= 0.5, μ (v,w)=0.6, μ(w,x)=0.7, μ(x,y)=0.8, μ(u,y) =0.5. Consider a Gc :(σ, μ) be fuzzy graph. Define σ(u) = 0.5, σ(v) = 0.6, σ(w) = 0.7, σ(x) = 0.8, σ(y) = 0.9, μ(u,x)= 0.2, μ(u,w) =0.2, μ(v,x)=0.2, μ(v,y)=0.2, μ(w,y)=0.2. Here G is complete fuzzy graph but in Gc, u is adjacent to vertices x and w with distincttotal degree. So $Gc$is not highly totally regular fuzzy graph.

Theorem 4.11

Let G* : (V,E) is even cycle. If σ takes same membership values and if μ is constant function or alternate edges have same membership values then G is highly totally regular.

Proof.

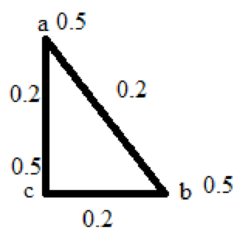
Case 1:

When μ is constant function. Since μ is constant function d(u) = k for all u Є V we have td(u) = d(u)+ σ(u) since σ takessame membership values there exists avertex v which is adjacent to u and w suchthat td(u) =td(w). Hence G is highly totallyregular fuzzy graph.

Case 2:

Alternate edges have same membership values Let $\mu(ei) = k1$when i is odd and $\mu(e) = k2$ when i is even. Let uv the edge with membership value $k1$ and vw be the edge with membership value $k2$. Since σ takes same membership values let σ(u) =$c1$ and σ(w) = $c2$,td(u)=$c1$+$k1$+$k2$ and td(w)=$c2$+$k1$+$k2$→td(u) = td(w) Hence G is highly totally regular fuzzy graph.

Example 4.12



Highly totally regular fuzzy graph (Fig 4.12)

Consider a G : (σ,μ) fuzzy graph. define σ(a) = 0.5, σ(b) = 0.5, σ(c) =0.5, μ(a, b) = 0.2,μ(y, z) = 0.2, μ(x, z) = 0.2or all u,v ЄE.

Theorem 4.13

Any highly totally regularfuzzy graph can have atleast one fuzzybridge.

Proof.

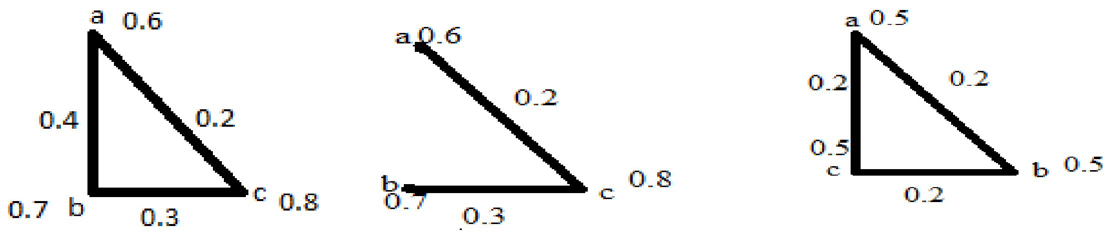
Case 1:

μ takes same membership values. Suppose uv be the edge with membership value $k1$ and vw be the edge with membership value $k2$such that $k1 = k2$ Then the removal of edge vw does not reduce the strength of connectedness v and w. Hence vw is not fuzzy bridge.

Case 2:

Alternate edges have same membership values Suppose alternate edges have distinct membership values such that $k1 < k2$ then the removal of the edge with highest membership value may reduce the strength of connectedness between those pair of vertices. Hence G can have fuzzy bridge.

Example 4.14



Fuzzy bridge (a b)The strength of connectedness Highly totally regular

Consider a G : (σ,μ) fuzzy graph. Define σ(a)= 0.6,σ(b) = 0.7, σ(c) =0.8, μ(a, b) = 0.4, μ(b, c) = 0.3, μ(a , c) = 0.2, here removal of edge a b reduce strength of connectedness. So a b is fuzzy bridge. Consider a H: (σ,μ) is regular fuzzy graph. So the removal of edge does not reduce strength of connectedness pairs of vertex.

Remark 4.15

The aboe theorem does not hold true if μ is constant function, Since the removal of any edge does not reduce the strength of connectedness of any pair of vertices

CONCLUSION.

In this paper the concept of highly regular and highly totally regular fuzzy graph discussed. Then comparative and necessary and sufficient condition are provided. Then Equivalent condition made .then some result established.

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