# Pseudo-Line Arrangements of Wiring Diagram

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**Abstract- Chamber sets are subsets of [n] that can be labeled with subsets of [n] given a wiring diagram (pseudo-line arrangement) of a permutation w $S_n$. In this brief note, we illustrate how two wiring diagrams (of the same w) can be mutated from one to the other using regular moves (derived from nil and braid relations), thus freezing the chamber sets they connect. We present a new approach for computing the graph of double wiring diagram commutation classes. We can confirm Fomin and Zelevinsky's positivity conjecture when n ≤ 4 by using these methods to compute the graph for five strings or less.**

***Keywords – Chamber sets, Pseudo-line arrangement, wiring diagram, Graph theory, Quiver diagram***

## I. INTRODUCTION

In the study of cluster graph theory, double wiring diagrams were introduced to study totally positive matrices and became a stimulating example. In particular, the graph showing the relationships between the commutation groups is a precursor to the ex- change graph, and the relationships between the minors of the chamber are precursors to exchange relationships.

The n-stringed dual cable diagram in Fomin and Zelevinsky should be defined as two sets of n partly linear linear lines (red and blue). Each line crosses every other line in the same colour exactly once. Red lines are numbered from 1 to n with 1 at left top and n at left bottom. In the reverse order, the blue lines are numbered. Furthermore, every cable chamber has a pair of subsets (r, b) in the cable map, where r (resp. b) is a subset of {1,2,....n} defining the red strings below the chamber (resp. blue).

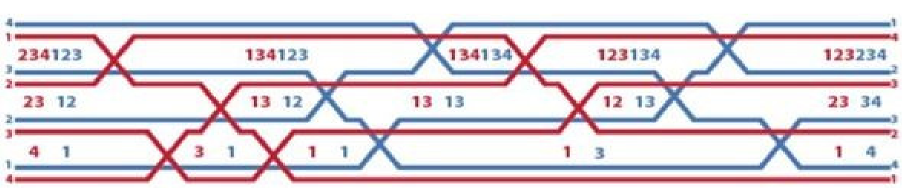


Figure-1.1

It is likely that the same set of chamber labels can generate slightly different wiring diagrams. Two wiring diagrams that share the same isotope set of chamber labels are taken into account by Fomin and Zelevinsky.

Such sets of diagrams are called commutation classes that have been studied in For single wiring diagrams, we use the term commutation classes of double wiring diagrams to keep this relation simple. A series of the braid moves pictured may connect any two commutation groups of double wiring diagrams. Notice that only one chamber mark moves in each exchange. Well name this mark the center of the motion of the braid.

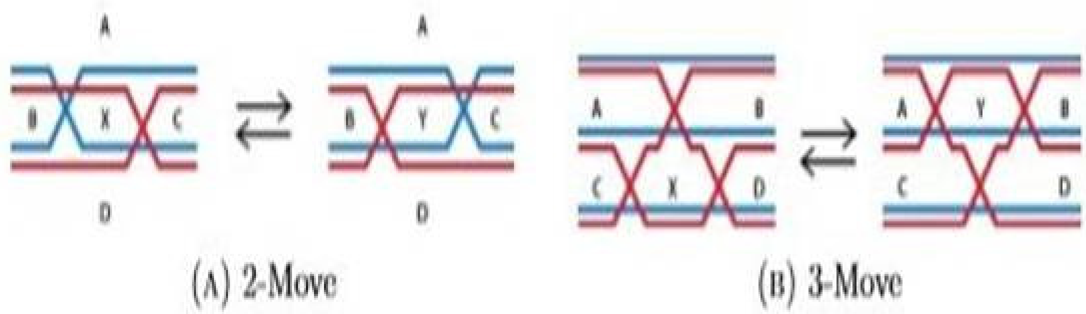


Figure-1.2.Braid Moves

## *1.1 A pseudo-line arrangement*

A pseudo-line may be simple non-contractible closed curve with in the projective plane. In particular, each line in the projective plane is a pseudo-line. Pseudo-lines and certain relationships between them inherit properties from the topological structure of the projective plane.

- Any two pseudo-lines have at least one point in common.
- If two pseudo-lines meet in exactly one point they intersect transversally at that point.

## II. ***PRELIMINARIES***

Cluster graph theory's were conceived by Fomin and Zelevinsky in the spring of 2000 as a tool for studying total positivity and dual canonical bases in Lie theory. However, the idea of cluster graph theory's has since taken on a life of its own, as connections and applications are discovered to diverse areas of mathematics, including quiver representations, tropical geometry, integrable systems, and Poisson geometry.

### *2.1 Definitions:*

#### 2.1.1 Definition

The graph of the wiring diagram commutation groups, $\varphi_n$, has a unique vertex for each double wiring diagram commutation class of n strings. Two vertices are connected by an edge if a single braid movement varies in their wiring diagrams.

Fomin and Zelevinsky demonstrate that $\varphi_n$ is a finite linked graph and calculation .We present a method for computing n in this article and use it to construct . We use these equations to test Fomin and Zelev's positivity conjecture.

#### 2.1.2 Definition

The quiver Q(w) has vertices corresponding to chamber labels and an arrow from (r,b) to $(r^1, b^1)$ if $(r^1) = rUr_j$ and $(b^1) = bUb_k$ for $r_jb_k$ 1,2,3. n shows Q(w) for the wiring diagram pictured in Figure 1.1 .

As appropriate we may label the arrows of the quiver with the pair of numbers being adjoined to r and b, or simply by the red or blue numbers individually.

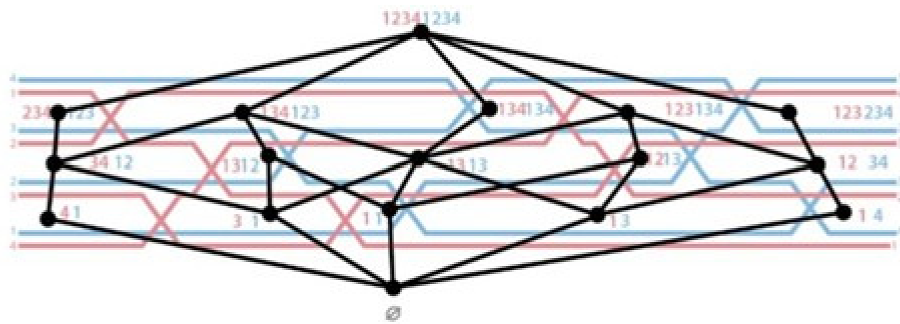

Figure 2.1 Quiver Diagram

**2.1.3 Definition**

A sub quiver is complete if every Q arrow connecting two vertices in the sub quiver is included in it.

**2.1.4 Definition**

A sub quiver is complete if it contains any Q vertex that lies within the sub quiver 's boundary.

**2.1.5 Definition**

A quiver is a directed multi graph with no loops or 2-cycles. The vertices are labeled with elements of [m]. A directed edge (i,j) will be denoted $i\ j$ A quiver mutation of a quiver Q at vertex j is a process, denned as follows, that produces another quiver $\mu_j(Q)$.

## III. *METHODOLOGY*

***3.1- THEOREM***

A label-centered 3-move exists (r, b) if and only if Q(w) contains one of the two complete, complete sub quivers shown in figure.

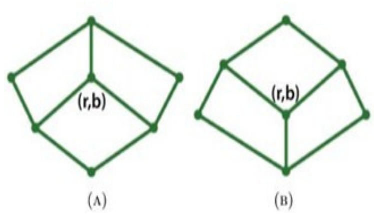


Figure- 3.1.1 Sub quivers for 3-move

Assume it there exists a 3-move. So there must be a region of the isomorphic wiring diagram in Figure1.2. The construction of the sub quiver from this diagram is shown in Figure 3.1.1.

Assume Q(w) has a full isomorphic compete sub quiver to Figure 4a. The possible red labels for this sub quiver are investigated. Since a path of length three connects the bottom vertex to the top, we know that only three separate edge labels can appear in this sub quiver. From the bottom to the top, we mark the leftmost path passing through (r, b) x, y, z. as pictured in Figure 3.1.2.

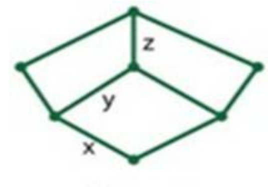


Figure 3.1.2

Only two different edge labels can be used for each four-cycle in the Figure 3.1.2 as the bottom and top vertices are connected by a path of length two. This limits the possible labels to those in Figure 3.1.3.

The case corresponding to picture d) does not occur in the figure because the strings z and y are exchanged twice, contradicting the concept of a diagram of double wiring.

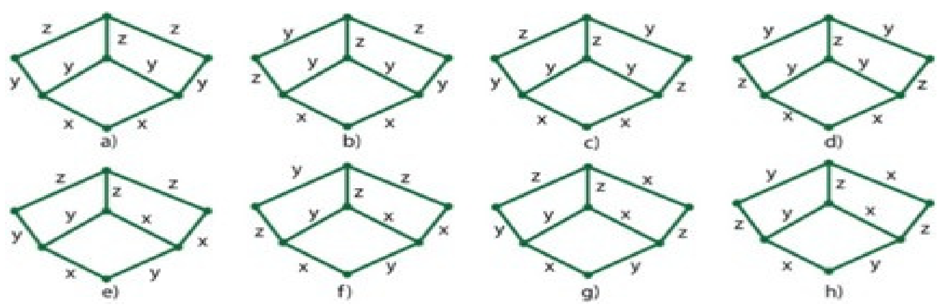


Figure 3.1.3 Labeled sub quivers

For the blue strings, repeat this statement and mark certain cases A through H. We are now deciding which red and blue cases can be mixed. The only potential pairs are (a, H),(b, G), and (c, F), and their opposites are (h, A),(g, B) and (f, C), since the labels must be different.

If, as in Figure3.1.4 we draw a sub quiver with the labels in the case (b, G), we retrieve an extra arrow that contradicts the assumption that the sub quiver was complete. The pairs (c, F), (g, B) and (f, C) are symmetrical to (b, G), thus replacing them as well. This only leaves (a, H) and (h, A) as potential names.

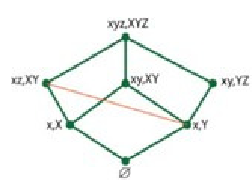


Figure 3.1.4

By symmetry of labels, the edge labels can be assumed to be form (h, A). Since this sub quiver is complete, no vertices are missing. Change in the cardinality's chamber label shows a uniquely crossing of the braid as shown in Figure 3.1.5(a).

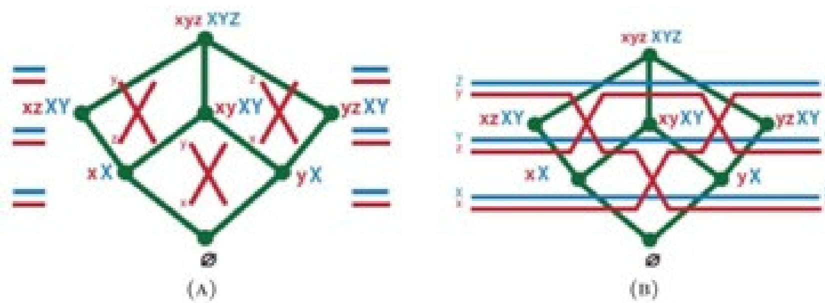


*Figure 3.1.5* Reconstructed 3-move

In this region, there can be no other crossings as the quiver is complete. Thus, with- out creating other crossings, strings must be connected. This gives the image of three movements.

### *3.2 THEOREM*

There exists a two move centered at label (r, b) if and only if Q(w) includes the required sub quiver shown here in Figure 3.2.1.

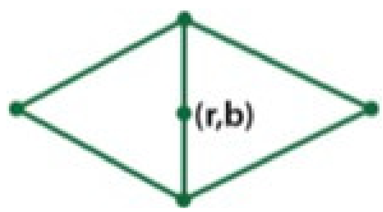


*Figure 3.2.1 Sub quiver for 2-move*

**PROOF:**

Assume that , two moves exist. A region of the wiring diagram isomorphic to Figure 1.2a must then be defined. The development of that quiver results in the sub quiver in Figure 3.2.1.

Assume that Q(w) contains the entire sub quiver in Figure 3.2.1. For the sub quiver, we look at possible red labels. Mark the arrows of (r, b) to and from x and y. Since there's a path from rock bottom vertex to the highest vertex of length two, all arrows within the sub quiver must be labeled x or y. The possible labels are shown in Figure 3.2.2. The case of figure d) can be eliminated because strings x and y must be exchanged twice.

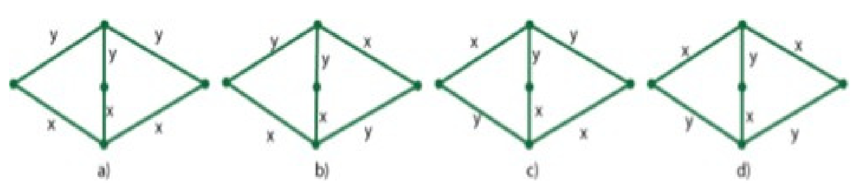


Figure 3.2.2

We build a similar pattern for blue strings by labeling the0 X and Y arrows. What red and blue cases can be paired together, we must determine. The only potential cases pairs are (b, C) and (c, B) because all the labels are distinct (see Figure 3.2.3).

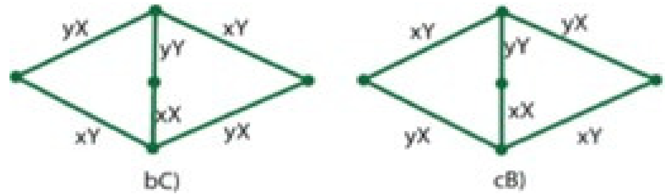


Figure 3.2.3

Since the labels are symmetrical, the diagram can be assumed to have edge markers of type (b ,C) without loss of generality. There are no missing vertices since this sub quiver is complete. This indicates an unique crossing in changes to the chamber labels of the same cardinality of the braid as shown in Figure 3.2.4(a).

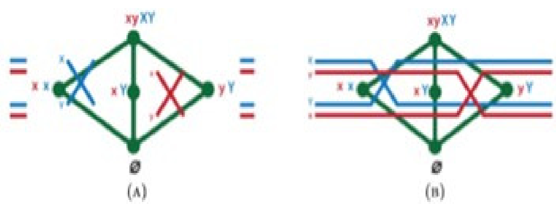


Figure 3.2.4

Since no more crossings can happen in this region, without any other crossings, we connect the strings. Which results in Figure 3.2.2b, which is also in 2 moves.

## IV.CONCLUSION

A method for computing the graph of wiring diagram commutation classes is presented. Using these methods to compute the graph for five strings or less. In this paper "PSEUDO LINE ARRANGEMENTS OF WIRING DIAGRAM" we conclude that double wiring diagram has same connected moves in every crossing's of the quiver diagrams. Thus quiver diagram and sub quiver diagram has number of strings where it passes in a same region.

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