

Implementation of IVAM method and Stepping Stone method to get an optimal solution for Triangular fuzzy numbers.

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ABSTRACT-In this paper, we have made an implementation of Improved Vogel's approximation method and stepping stone method in fuzzy transportation problem. There are several approaches to solve these types of problems, where the Vogel's approximation method and its modification are the best ways to find the lowest cost. A variant of VAM was proposed by using total opportunity cost matrix. IVAM method is a modified method of VAM. Here, the initial feasible solution is derived by using fuzzy improved Vogel's approximation method and stepping stone method. A numerical method is illustrated. Here the results give that stepping stone method gives best result.

Key words: Improved Vogel's approximation method, fuzzy transportation problem, Stepping stone method.

I. INTRODUCTION:

A transportation problem permits shipments that go directly from a point of supply to a point of demand. Shipments are allowed between supply or between demand in certain cases. There may also be points (called transshipment points) from which goods may be transported from a supply to a demand point or in their journey. Shipping issues are transshipment issues with any or all of these functions. Both parameters are fuzzy numbers in Fuzzy Transport. Standard or irregular, triangular or trapezoidal numbers can be fuzzy numbers. The Mean of Maxima method has been implemented on the basis of this theory, transforming the issue of fuzzy transportation. The idea is to turn a problem with fuzzy parameters into change for fuzzy number into crisp form.

Decision parameters such as availability, necessity and unit transport cost of the model must be set at crisp values in order to solve a transportation problem. But, due to many factors, supply, demand and unit transportation costs can be unpredictable in real life applications. Fuzzy numbers can represent such imprecise data. In 1965, Zadeh [17] introduced the concept of the fuzzy package. Bellmann and Zadeh [2] suggested the idea of decision making. Chanas and Kuchta [2] have suggested a method for solving the problem of fuzzy transport. A system for solving fuzzy transport problems has been developed by Chanas et al [4]. Liu and Kao [8] suggested a new approach to solve the issue of fuzzy transportation by using Zadeh's Extension principle.

In section 2, we recall the concepts of triangular fuzzy numbers. In section 3 we see about fuzzy transportation problems and some results. In section 4, we have introduced fuzzy transportation problems and solving them using IVAM and simplifying using stepping stone method. In this paper we propose a new algorithm to find the optimal solution to the maximum method of a fuzzy transport problem, which is used to change the fuzzy number into a crisp shape. In the end, it is possible to achieve the optimal solution of a problem in a fuzzy number or a crisp number.

II. PRELIMINARIES:

2.1 *Fuzzy set*: A fuzzy set is characterised by a membership function mapping element to the unit interval $[0, 1]$ of a domain, space, or universe of discourse X . Fuzzy set is a pair (V, μ) where V is a set and $\mu: V \rightarrow [0, 1]$ is a membership function.

2.2 *Fuzzy number*: A fuzzy number is a generalisation of a regular, real number in the sense that it does not apply to a single value, but to a set of possible values connected, where each possible value has a weight of its own between 0 and 1. This weight is referred to as the membership function. Thus a fuzzy number is a special case of the real line of a convex, normalised fuzzy set.

2.3: *Triangular fuzzy number*: It is like a triplet. Example (x, y, z) is a triangular fuzzy number, x, y and z are real numbers. $x < y < z$. Here x is the smallest number, y is the probable number and z is the largest number.

2.4 *Arithmetic operations in triangular fuzzy numbers*: In general, the fundamental arithmetic operations between Fuzzy numbers are carried out in two alternative ways

- i. With the help of the α -cuts of the corresponding FNs, which, as we have already shown, are the normal closed \mathbb{R} intervals. The Fuzzy Arithmetic, therefore is simply based on the arithmetic of the real intervals according to this method.
- ii. Applying the Zadeh extension principle which provides the means to generalise any function f mapping the crisp set X to the crisp set Y so that fuzzy subsets X can be mapped to fuzzy subsets Y . The above two general methods of fuzzy arithmetic, which involve laborious calculations, are rarely used in applications where the use of simpler types of FNs, like TFNs, is preferred.
- iii. The above two general methods can be seen to lead to the following basic rules for adding and subtracting Triangular fuzzy numbers.

Take triangular fuzzy sets $A = (x, y, z)$ and $B = (x_1, y_1, z_1)$ be two TFNs. Then

- i. The sum of $A + B = (x+x_1, y+y_1, z+z_1)$.
- ii. The difference of $A - B = A + (-B) = (x-z_1, y-y_1, z-x_1)$, where $-B = (-z_1, -y_1, -x_1)$ is defined to be the *opposite* of B .

In other words, triangular fuzzy number are both the opposite of a TFN, as well as the number and variance of the two triangular fuzzy numbers. On the contrary, the product and the quotient of two triangular fuzzy numbers are not always triangular fuzzy numbers, whereas they are fuzzy numbers. However in the special case where x, y, z, x_1, y_1, z_1 are in \mathbb{R}^+ , it can be can be approximately performed by the rules:

- iii The product of $A \cdot B = (x \cdot x_1, y \cdot y_1, z \cdot z_1)$
- iv The quotient of $A/B = (\frac{x}{x_1}, \frac{y}{y_1}, \frac{z}{z_1})$

In other words, the inverse of a Triangular fuzzy number, as well as the product and the division of two Triangular fuzzy numbers, may also be considered roughly to be Triangular fuzzy number in \mathbb{R}^+

2.5 *Defuzzification*: Defuzzification is the method of transforming, with respect to a fuzzy set, a fuzzified output into a single crisp value. In FLC (Fuzzy Logic Controller), the defuzzed value represents the action to be taken to control the operation.

2.6 *Mean of Maxima method*: The defuzzified value is taken as the variable that has the highest membership values in this process. If there is a maximum membership value of more than one variable, the mean value of the maximum is taken. Let A be a fuzzy set with a function of membership $\mu_A(x)$ defined over $x \in X$ a universe of discourse. A fuzzy set x^* is defined as the defuzzified value and is defined as

$$x^* = \frac{\sum_{x_i \in M} \mu_A(x_i)}{|M|}$$

Here, $M = \{x_i | \mu_A(x_i) = \max_{x \in X} \mu_A(x)\}$, The height of the fuzzy set A is equal to and the cardinality is $|M|$ of set M

2.7 Fuzzy transportation problem: Assume a situation with m sources or supply centres containing different quantities of product to be assigned to n destinations or demand centres. Since transportation from any one source to any one destination is possible, The problem is that the number of units to be transported from origin i to destination j is calculated in such a way that all specifications are met at the lowest overall cost of transportation. For a balanced transport problem, this scenario holds. In addition, the amount of availability or supply of the origin is not equal to the sum of the requirements in an unbalanced transport problem

III. METHODOLOGY:

The Transportation Table now looks like a fuzzy transport table in a fuzzy environment. We now use Fuzzy Vogel's Approximation Method to solve the fuzzy transportation problem. Here to find the big or small fuzzy numbers, we use a Mean of maxima method to defuzzify the fuzzy numbers. All the cost, supply and demand of the problem of fuzzy transportation are fuzzy here So first we convert the fuzzy cost to #level cost and then apply the approximation method of Vogel, we get the initial fuzzy solution and then we apply the enhanced approximation method of Vogel for the same problem

3.1 Fuzzy Vogel's Approximation method algorithm:

- i. Balance the given transportation problem
- ii. Identify the smallest and next to the lowest cost for each row of the fuzzy Transportation table (do not consider the diagonal zeros). For each row, determine the difference between them and show them along the side of the fuzzy Transportation table by enclosing them against the respective rows in parenthesis. Likewise, measure the difference for each column and display it in parenthesis at the bottom of the table
- iii. Identify the row or column with the largest number in the parenthesis of all the variations shown on the side and bottom of the table. Use any arbitrary tie-breaking option if a tie exists. Let the greatest differentiation refer to the i th row and let the lowest cost in the i th row be c_{ij} . Allocate the maximum possible fuzzy amount $x_{ij} = \min(a_i, b_j)$ th cell and cross either the i th row or the j th column in the normal manner (give diagonal zero first preference).
- iv. For the reduced fuzzy Transportation table and goto phase (ii), the column and row differences were re computed. Repeat the procedures until the full criteria for the rim are met.
- v. Calculate the overall cost of fuzzy transportation using the initial balanced fuzzy transportation cost matrix for the feasible cost for the feasible allocations.

3.2 Fuzzy Improvised vogel's approximation method:

By using the total opportunity cost matrix (TOC) and alternative allocation costs. Van has been enhanced. The TOC matrix is obtained by adding row opportunity cost matrix and column opportunity cost matrix.

Row opportunity cost matrix: The smallest cost of that row is subtracted from every element of the same row for each row.

Column opportunity cost matrix: The smallest cost of that column is subtracted from every element of the same column for each column

3.2.1 Fuzzy Improvised vogel's approximation method algorithm:

- i. Balance the transportation problem provided
- ii. Find the TOC matrix.
- iii. Identify boxes with minimum and next to minimum transport costs in each row and write the difference (penalty) against the corresponding row along the side of the table
- iv. In each column, mark the boxes with minimum and next to minimum transport costs and write the difference (penalty) against the corresponding column.
- v. Identify the maximum penalty. If it is along the side of the table, make the full allocation in that row to the box with minimal transportation costs. If it is below the table, make the maximum allocation to the box in that column with the minimum transport cost.
- vi. If the penalties for two or more rows or columns are equal, pick the topmost row and the leftmost column, respectively
- vii. Select the topmost row and the leftmost column, respectively, if the penalties for two or more rows or columns are equal.

- viii. Calculate the new penalty cost as step 3 for the remaining submatrix and assign it following the previous step process. Continue the procedure to satisfy all rows and columns.
- ix. Calculate the overall cost of fuzzy transport for the feasible costs for the feasible allocations using the initial matrix of balanced fuzzy transshipment costs.

3.3 The Stepping Stone method to find optimal solution for fuzzy transportation problem:

- i. Determine the initial feasible solution using improvised vogel’s approximation method.
- ii. Make sure the number of cells occupied is exactly equal to $m+n-1$, where m is the number of rows and n is the number of columns.
- iii. This testing of each unoccupied cell is conducted by the following five steps as follows:
- iv. Select an unoccupied cell.
- v. Starting with this cell, trace a closed path through at least three horizontal and vertical motions using the most direct route. In addition, since only the cell is used in the solution at the turning point and then back to the original occupied cell and continuing on the closed path, both unoccupied and occupied boxes can be skipped over. Stepping stones on the road are called the cells at the turning points.
- vi. Alternatively, add plus(+) and minus(-) signs to each corner cell of the closed path just traced, starting with the plus sign to be evaluated at the unoccupied cell.
- vii. Calculate the net cost adjustment along the closed path by integrating the unit cost in each square containing the minus sign.
- viii. Repeat sub-step (a) via sub step (b) until all unoccupied cells of the transport table have been determined with a 'net change in cost'.
- ix. Check each of the net changes for signs. If all measured net changes are greater than or equal to zero, an optimum solution is found. If not the current solution could be improved and total shipping costs reduced.
- x. Determine the maximum number of units that can be assigned to a cell marked with a minus sign on the closed path corresponding to that cell by selecting the unoccupied cells with the greatest negative net cost shift. Add this number to the unoccupied cell and all other cells indicated by a plus sign on the road. Subtract this number from the cells marked with the minus sign on the closed path.
- xi. Now Go to step 2 and repeat the process until we reach an optimal solution.

IV. NUMERICAL ILLUSTRATION:

Consider the following fuzzy transportation problem which is in , All of data in this problem are triangular fuzzy numbers. The example was solved by the method proposed by the authors of this paper. Afterwards, the results obtained are compared

Table 1

	D_1	D_2	D_3	D_4	Supply
S_1	(17,18,19)	(13,14,15)	(4,5,6)	(13,14,15)	(250,350,450)
S_2	(3,4,5)	(5,6,7)	(3,4,5)	(4,5,6)	(150,250,350)
S_3	(19,20,21)	(21,22,23)	(10,11,12)	(23,24,25)	(500,600,700)
S_4	(10,11,12)	(14,15,16)	(2,3,4)	(17,18,19)	(500,600,700)
Demand	(200,300,400)	(300,400,500)	(350,450,550)	(550,650,750)	

Solution:

Using mean of Maxima method we need to find the crisp values for triangular fuzzy numbers.

Table 2

	D_1	D_2	D_3	D_4	Supply
S_1	18	14	5	14	350
S_2	4	6	4	5	250
S_3	20	22	11	24	600
S_4	11	15	3	18	600
Demand	300	400	450	650	

Now we apply improved vogel’s approximation method,

We have to find TOC matrix

First, we have to find row opportunity cost table

Table 3

	D₁	D₂	D₃	D₄
S₁	13	9	0	9
S₂	0	2	0	1
S₃	9	11	0	13
S₄	9	12	0	15

Table 4

	D₁	D₂	D₃	D₄
S₁	14	8	2	9
S₂	0	0	1	0
S₃	16	12	8	19
S₄	7	9	0	13

The Total Opportunity Cost matrix is,

Table 5

	D₁	D₂	D₃	D₄	Supply
S₁	27	17	2	18	350
S₂	0	2	1	1	250
S₃	25	23	8	32	600
S₄	16	21	0	28	600
Demand	300	400	450	650	

Table 6

	D₁	D₂	D₃	D₄	Supply	Row penalty	
S₁	27	17	2	18(350)	350	15 15 1 -- -- -- --	
S₂	0	2	1	1(250)	250	1 -- -- -- -- -- --	
S₃	25(150)	23(400)	8	32(50)	600	15 15 2 2 2 2 23	
S₄	16(150)	21	0(450)	28	600	16 16 5 5 -- -- --	
Demand	300	400	450	650			
Column penalty	16 9 9 9 25 25 --	15 4 4 2 23 23 23	1 2 -- -- -- -- --	17 10 10 4 32 -- --			

The initial feasible solution is obtained.

The minimum transportation cost = $18 \times 350 + 1 \times 250 + 25 \times 150 + 23 \times 400 + 32 \times 50 + 16 \times 150 + 0 \times 450 = 23500$

the number of allocated cells = 7 is equal to $m + n - 1 = 4 + 4 - 1 = 7$

Optimality test using stepping stone method. First iteration for optimum test:

Step1: Create a closed loop for unoccupied cells.

Table 7

Unoccupied cell	Closed path	Net cost change	Result
S1D1	S1D1→S1D4→S3D4→S3D1	27 - 18 + 32 - 25=16	Cost increase
S1D2	S1D2→S1D4→S3D4→S3D2	17 - 18 + 32 - 23=8	Cost increase
S1D3	S1D3→S1D4→S3D4→S3D1→S4D1→S4D3	2 - 18 + 32 - 25 + 16 - 0=7	Cost increase
S2D1	S2D1→S2D4→S3D4→S3D1	0 - 1 + 32 - 25=6	Cost increase
S2D2	S2D2→S2D4→S3D4→S3D2	2 - 1 + 32 - 23=10	Cost increase
S2D3	S2D3→S2D4→S3D4→S3D1→S4D1→S4D3	1 - 1 + 32 - 25 + 16 - 0=23	Cost increase
S3D3	S3D3→S3D1→S4D1→S4D3	8 - 25 + 16 - 0=-1	Cost decrease
S4D2	S4D2→S4D1→S3D1→S3D2	21 - 16 + 25 - 23=7	Cost increase
S4D4	S4D4→S4D1→S3D1→S3D4	28 - 16 + 25 - 32=5	Cost increase

Step2: Select the unoccupied cell having the highest negative net cost change i.e. cell $S_3D_3=-1$.and draw a closed path from S_3D_3 .Closed path is $S_3D_3 \rightarrow S_3D_1 \rightarrow S_4D_1 \rightarrow S_4D_3$ Closed path and plus/minus allocation for current unoccupied cell S_3D_3

Table 8

	D_1	D_2	D_3	D_4	Supply
S_1	27	17	2	18(350)	350
S_2	0	2	1	1(250)	250
S_3	25(150)[-]	23(400)	8[+]	32(50)	600
S_4	16(150)[+]	21	0(450)[-]	28	600
Demand	300	400	450	650	

Step3: Minimum allocated value among all negative position (-) on closed path = 150
Substract 150 from all (-) and Add it to all (+)

Table 9

	D_1	D_2	D_3	D_4	Supply
S_1	27	17	2	18(350)	350
S_2	0	2	1	1(250)	250
S_3	25	23(400)	8(150)	32(50)	600
S_4	16(300)	21	0(300)	28	600
Demand	300	400	450	650	

Now we have to repeat the step 1 to 3, to get an optimal solution is obtained.

Second iteration for optimal test

Create a closed loop for unoccupied cells.

Table 10

Unoccupied cell	Closed path	Net cost change	Result
$S1D1$	$S1D1 \rightarrow S1D4 \rightarrow S3D4 \rightarrow S3D3 \rightarrow S4D3 \rightarrow S4D1$	$27 - 18 + 32 - 8 + 0 - 16 = 17$	Cost increase
$S1D2$	$S1D2 \rightarrow S1D4 \rightarrow S3D4 \rightarrow S3D2$	$17 - 18 + 32 - 23 = 8$	Cost increase
$S1D3$	$S1D3 \rightarrow S1D4 \rightarrow S3D4 \rightarrow S3D3$	$2 - 18 + 32 - 8 = 8$	Cost increase
$S2D1$	$S2D1 \rightarrow S2D4 \rightarrow S3D4 \rightarrow S3D3 \rightarrow S4D3 \rightarrow S4D1$	$0 - 1 + 32 - 8 + 0 - 16 = 7$	Cost increase
$S2D2$	$S2D2 \rightarrow S2D4 \rightarrow S3D4 \rightarrow S3D2$	$2 - 1 + 32 - 23 = 10$	Cost increase
$S2D3$	$S2D3 \rightarrow S2D4 \rightarrow S3D4 \rightarrow S3D3$	$1 - 1 + 32 - 8 = 24$	Cost increase
$S3D1$	$S3D1 \rightarrow S3D3 \rightarrow S4D3 \rightarrow S4D1$	$25 - 8 + 0 - 16 = 1$	Cost increase
$S4D2$	$S4D2 \rightarrow S4D3 \rightarrow S3D3 \rightarrow S3D2$	$21 - 0 + 8 - 23 = 6$	Cost increase
$S4D4$	$S4D4 \rightarrow S4D3 \rightarrow S3D3 \rightarrow S3D4$	$28 - 0 + 8 - 32 = 4$	Cost increase

The net cost change for the values is positive. So final optimal solution is arrived.

Table 11

	D_1	D_2	D_3	D_4	Supply
S_1	27	17	2	18(350)	350
S_2	0	2	1	1(250)	250
S_3	25	23(400)	8(150)	32(50)	600
S_4	16(300)	21	0(300)	28	600
Demand	300	400	450	650	

The minimum total transportation cost = $18 \times 350 + 1 \times 250 + 23 \times 400 + 8 \times 150 + 32 \times 50 + 16 \times 300 + 0 \times 300 = 23350$

V.CONCLUSION:

In this paper, we have proposed a new algorithm for the optimal fuzzy solution to the given fuzzy transportation problem with triangular fuzzy number converted into crisp transportation problem using mean of maxima method indices and the Stepping Stone method has been applied to find an optimal solution

for fuzzy numbers. First, we can use the fuzzy Improved Vogel's Approximation as the initial solution to the fuzzy transport problem.

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