DIAGNOSING ECG SIGNALS USING SINGULAR VALUE DECOMPOSITION

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***Abstract* - Heart condition diagnosis based on ECG signal analysis is the basic method used in prevention of cardiovascular diseases. Singular Value Decomposition which is regarded as a high resolution spectrum estimation tool decomposes the ECG data matrix into orthogonal subspaces corresponding to the signal and noise components contained in ECG data.Here Singular Value Decomposition is used as a tool for ECG signal de-noising through simulated and recorded data. The corrupting noise and the actual ECG signal lived in the dominant and subdominant subspaces suggests that this method is very effective for problems especially when we make a good choice of the projection operator.**

Keywords- Electro-Cardiography, Singular Value Decomposition, Denoising, ECG Signal and enhancement.

I. INTRODUCTION

ECG, an indicator of cardio bioelectrical activities is one of the most important criterion, helps physician and doctors in recognizing hard diseases and also someone's health. An Electrocardiogram describes the electrical activity of the heart which is decomposed into components of P,Q,R,S and T waves. When an ECG is recorded, it is contaminated with many kinds of noises. Therefore, extraction of pure cardiological indices from noisy measurements has been one of the major problems of biomedical signal processing and needs reliable techniques to preserve the diagnostic information of the recorded signal.

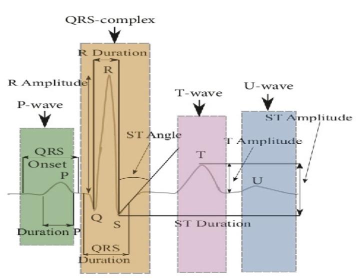

Figure 1: Heartbeat pulse model

II. SINGULAR VALUE DECOMPOSITION

The method of SVD is an important tool in signal processing and analysing the statistical data. SVD is representing signal in time-frequency domain. If there is a signal and it's possible to convert it into m x n matrix, SVD matrix can be written as below:

$$X = USV^T$$

Here Um x m and Vn x n respectively are called left and right singular value matrix. Also S is a diagonal matrix m x n includes singular values which the elements on main diagonal ofit are non-zero and the rest of the elements are zero. Members of the main diagonal of this matrix are like $\sigma_{11} > \sigma_{22} > \cdots > \sigma_{rr} > 0$ that are so called as the singular values of matrix data of x. This shows the Singular values of the signal information like noise level, the amount of signal energy, the number of elements that makes the signal. Although, the singular values also present the importance and the location of singular values in a matrix. From this we say, the greater the singular values, themore important the corresponding singular values. Of the most important usages of SVD we can, signal noise reduction and element separation that on this purpose we used singular values and the property of singular vectors perpendicular. Singular values of several different signals can be the same. That is why this parameters of signal processing cannot be reliable. The property of singular vectors being perpendicular is written as,

$$UU^T = V^TV = I$$

I is square matrix of unit and V^T is the transposed matrix of V. As we said, we use SVD to reduce noise. When the signal is highly co-related and it is mixed with one dimensional noise using SVD for mapping toa higher dimension space, the data that is more co-related means the dataof the main signal that is aligned in one direction. This means that singular values related to the ECG are greater and the singular values of noise are assigned to smaller values which are separated from each other by singular values. As a result, we can separate the noise signal from the main signal. Therefore, the singular values of the signal and noise cannot be perfectly separated from each other. We follow the separating procedure as follows;

$$X = USV^T = \begin{pmatrix} U_a & U_{a,n} & U_n \end{pmatrix} \begin{pmatrix} S_a & 0 & 0 \\ 0 & S_{a,n} & 0 \\ 0 & 0 & S_n \end{pmatrix} \begin{pmatrix} V_a^T \\ V_{a,n}^T \\ V_n^T \end{pmatrix}$$

Here the 'a' indices for main signal and 'n' indices for noise. The above matrix describes the singular values of signal and noise isadded in the number of the matrix elements. Here, the method used in filtering the singular values of SVD mapping presents to omit noise from the ECG signal. In this process, at first the signal is transferred to the higher dimension space by using SVD mapping. Therefore, using an algorithm the singular values are optimized through the reverse mapping.

III. METHOD OF SIGNAL ENHANCEMENT

In this procedure, after applying SVD on noisy signal, the singular values matrix is achieved. The values are as below:

$$S = \begin{pmatrix} S_a & 0 & 0 \\ 0 & S_{a+n} & 0 \\ 0 & 0 & S_n \end{pmatrix}$$

In which, S_{a+n} is a diagonal matrix contains the combination of singular values of the clean signal and noise signal, whose values are added together. In order to separate these values from each other, in this work a single function is proposed that is applied on the singular values of noisy signal, their values can approach to the singular value of the clean signal. On the other hand, the singular value of noise signal are omitted from the singular value matrix by applying the proposed function and in this way, the singular value of noisy signal will be optimized. Therefore, by the inverse SVD upon the optimized singular value and the right and left singular vectors of noisy signal, that segment of the signal will be enhanced and rebuilt. This process continues till the end of the signal. While, whenever required, zero padding method is used. The proposed equation is as below:

$$\alpha_i = \begin{cases} 1 & 1 \le i \le 3 \\ e^{-(i-4)/4.5} & 4 \le i \le 15 \\ 0 & 16 \le i \le 40 \end{cases}$$

In which , α_i is a coefficient that whenever is multiplied into the singular value of i^{th}element of noisy signal, its value will be enhanced. The number of singular values in singular value matrix are based on the chosen length of the window and it is achieved to be 40 which results in the number of singular values also to be 40. So the proposed method for 40 coefficients is considered.

The use of subspaces approach can enhance ECG signals corrupted by white Gaussian noise for short data records. A signal-plus noise model can be given by:

t=s+n

Where t is the noised signal, s is the clean signal and n the added noise.

The signal estimates s.... and noise estimate n.... may be obtained as,

$\hat{s} = P_r t$ and $\hat{n} = (I - P_r)t$

Where $I - P_r = P_r^{\perp}$ and,

$$t = [t(1)t(2) \ldots\ldots t(n)]^t$$

IV. ECG DYNAMIC MODEL

4.1 Methods And Results

The MIT-BIH Atrial Fibrillation Database was used to study the performance of the presented method. This database was recorded at a sampling rate of 128 Hz from 18 subjects with no significant arrhythmias.

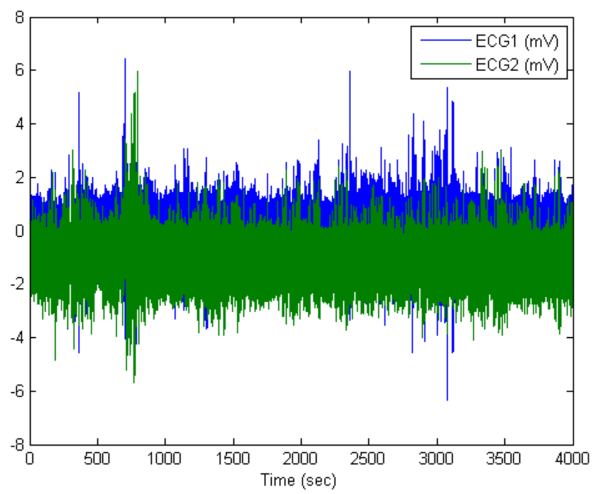

Figure 2 : Record afdb/ 04015

This database includes 25 long-term ECG recordings of human subjects with atrial fibrillation. Of these, 23 records include the two ECG signals and the individual recordings are each 10 hours in duration, and contain two ECG signals each sampled at 250 samples per second with 12-bit resolution over a range of ±10 millivolts.

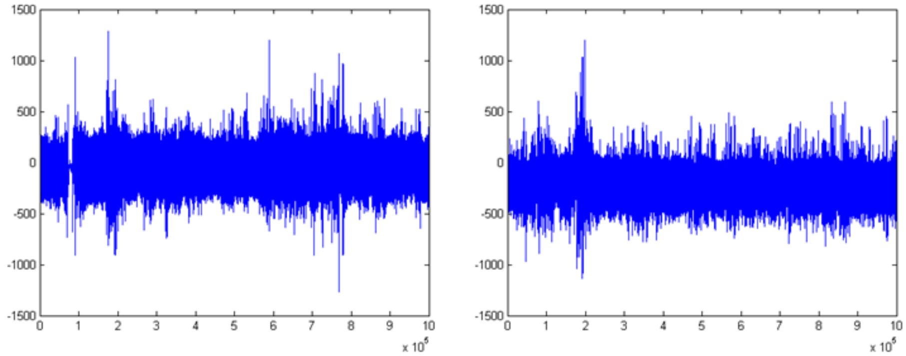

Figure 3 :Splitting up of ECG1 and ECG2 from Record afdb/ 04015

Since ECG signals are contaminated with noise, We apply SVD on noisy signal and denoising is done by applying the inverse SVD upon the optimized singular value and the right and left singular vectors of noisy signal so that the segment of the signal will be enhanced and rebuilt.

4.2 Measurement Criteria:

For evaluating the performance of the method, we have used the signal to noise ratio (SNR) Which is given by,

$$SNR = 10\ log_{10}\left(\frac{\sum x_{org}^2}{\sum\left(x_{org} - x_{est}\right)^2}\right)$$

Here , x_{org} indicates the clean signal and x_{est} indicates the enhanced one. We also investigate the time-frequency graph in addition with SNR.

Ten signals are selected from Physiobank ATM standard database which are usually mixed with the white additive noise and is applied five times for each method and average is taken out. The SNR varied from 11dB to 20dB and the average results are given below:

TABLE - I SNR of ECG enhancement by proposed method

S.No	INPUT SNR	OUTPUT SNR
1	6.8	11.30
2	9.29	13.85
3	12.81	16.70
4	15.83	19.60

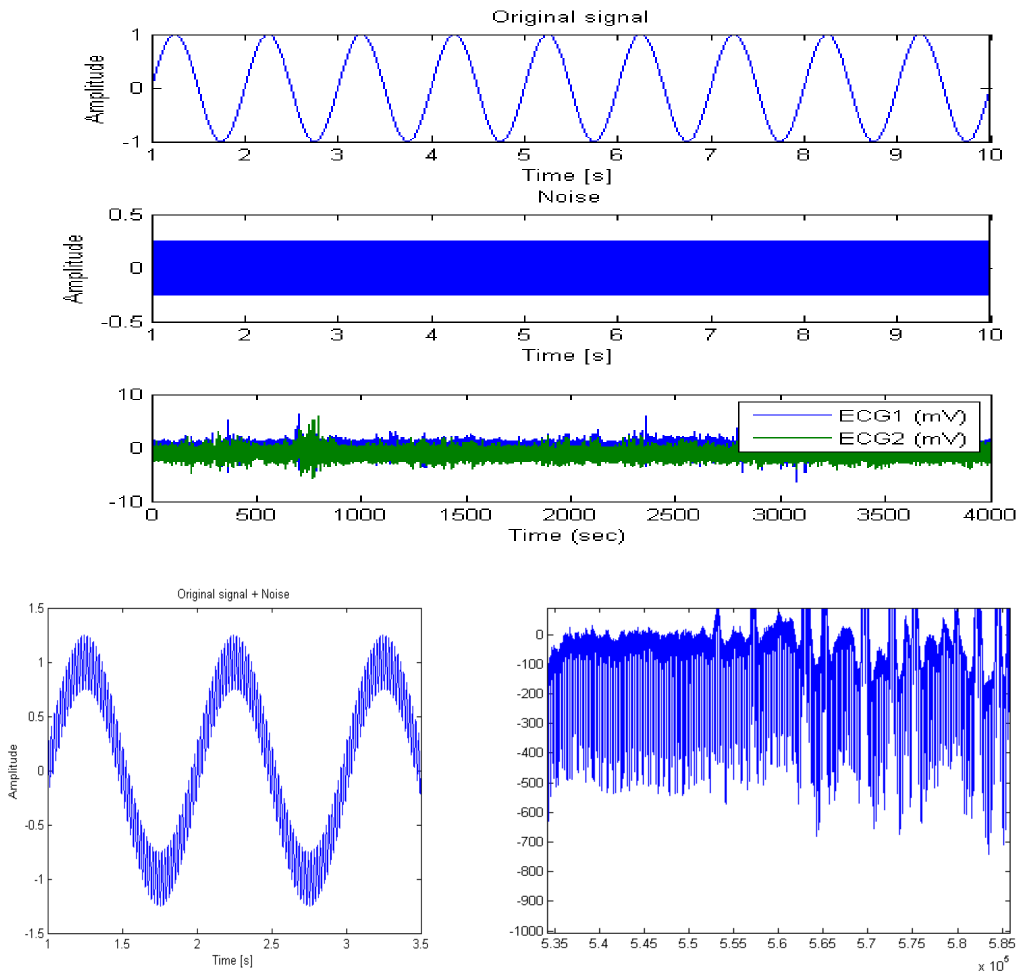

Figure 4: Noised Synthetic ECG Signal

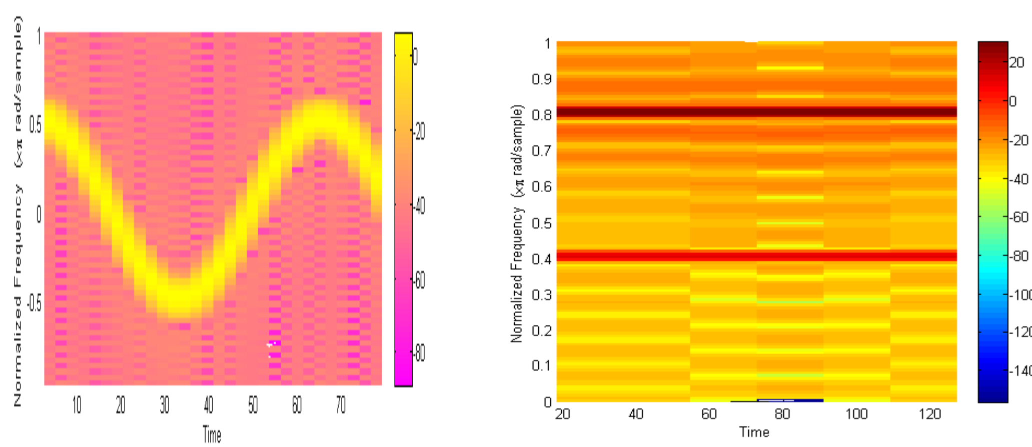

Figure5: Investigation of time- frequency graph

V. CONCLUSION

We apply SVD on noisy signal and obtain the singular value matrix which is a combination of singular values of the clean signal and noise signal. In order to separate these values, we propose a single function and apply on the noisy signal. Their values approaches to the singular value of the clean signal so that the singular value of noise signal are omitted from the singular value matrix by applying the proposed function. Using this method, the singular value of noisy signal will be optimized. Therefore by the inverse SVD upon the optimized singular value and the right and left singular vectors of noisy signal, the segment of the signal will be enhanced and rebuilt.

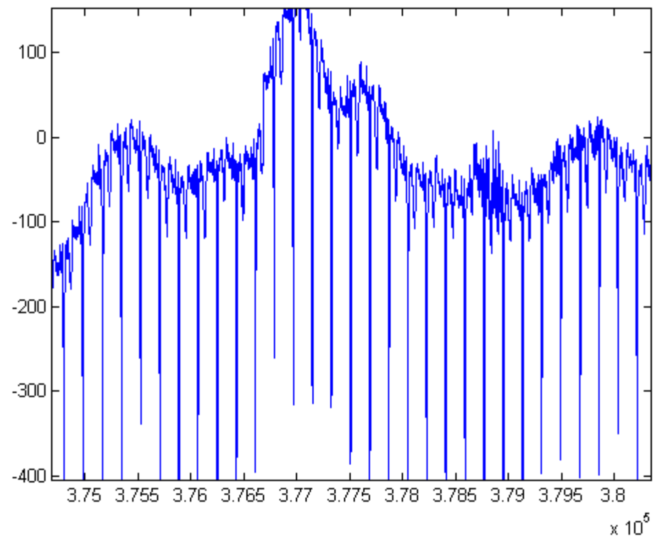

Figure 6: ECG signal denoising in various samples

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