A STUDY ON MATHEMATICAL MYSTERIES OF 64 SQUARES

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Abstract

This paper investigates chess through the toolcalled Mathematics. Chess may be a complicated nonetheless simple to know game. The work of ErnstZermelo shows that one player ought to be ready to force a win or force a draw. The work of Shannon and Hardy reveals the complexities of the sport. Probability, Combinatorics, and a few chess puzzles are utilised to higher perceive the sport. A computer is employed to check a hypothesis concerning chess strategy. Through the utilization of this program, we have a tendency to see that it's harmful to be the primary player to lose the queen. Ultimately, it's shown that mathematics exists inherently in chess. So maths is wont to improve, however not to excel, chess skills.

KEY WORDS: Chess, Mathematics,Zermelo, Probability,combinatorics,Hardy, Shannon.

Introduction

The game of chess has been around for hundreds of years. It has played a prominent role in Western culture since the Middle Ages.The game takes strategy, patience, and above all, problem solving. This study will attempt to explain how mathematics, particularly probability, statistics, and combinatorics, fits into the world of chess.

The game is played on a square board made up of 64 smaller squares. Half of these small squares are black and the other half are white, making a checkerboard pattern. At the beginning of the game, there are 32 pieces, 16 black and 16 white. Each player has eight pawns, two castles, two knights, two bishops, one queen, and one king.

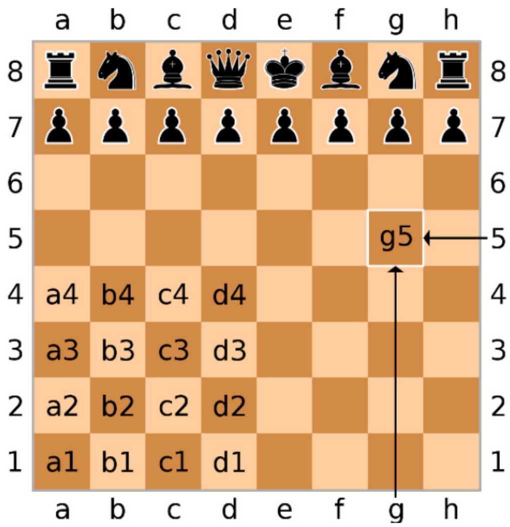
For the purposes of this paper, we will refer to castles as rooks from now on. It may move two spaces forward or backward, then one space to the left or right. It can also move two spaces left or right, then one space forward or backward. The knight is the only piece that can "jump" over other pieces to reach its destination. The bishop may move any number of spaces diagonally. Furthermore, the bishop that begins on a black square will never be on a white square for the duration of the game.

The pawn may move forward two squares on its first move, but may only move forward one square on all subsequent moves. The pawn can only move straight forward unless it is attacking an opposing piece.The queen is widely regarded as the most powerful piece on the board. It is essentially a combination of the rook and the bishop. The queen may move forward, backward, left, right, or diagonally as many squares as desired, until it meets a square already occupied by another piece.

The goal of chess is to capture the opposing player's king. When a king is in danger of being captured by an opposing piece, that king is in "check". If black makes a move to put the white king in check, white must, in its subsequent turn. This can be done by moving the king out of check, moving another piece to block the check, or capturing the piece that threatens the king. Whichever player forces checkmate wins the game.

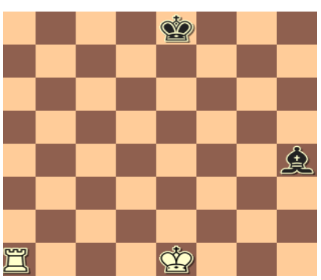
Notations are often used when writing about chess. Each piece is represented by a letter and each square on the board is represented by a combination of one letter and one number. These notations are demonstrated in the following table and image.

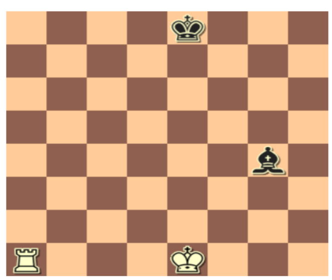
Piece	Notation	Symbol
King	K	♔
Queen	Q	♕
Rook	R	♖
Knight	N	♘
Bishop	B	♗
Pawn	P	♙

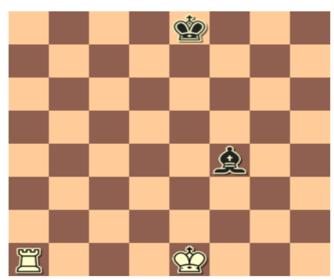


If the white player has his king on e1 and a rook on h1 with no pieces in between the two, he may choose to castle. He would then move his king to g1 and his rook to f1, only using up one turn. If castling is done using the rook on a1, the king would move to c1 and the rook would move to d1. Castling is extremely useful when trying to protect the king. Many chess games will see at least one of the players castle at some point.

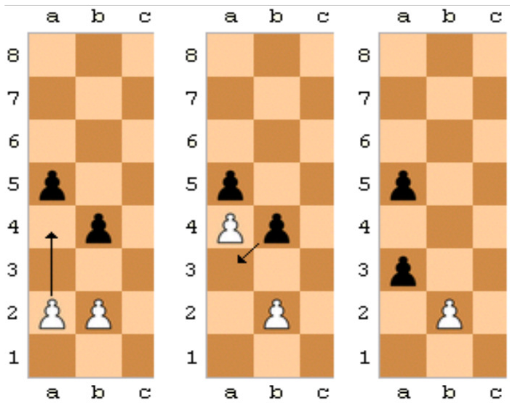
A player may castle only if neither his king nor his rook have moved from their starting position. A king cannot castle through check, out of check, or into check. So white would not be able to castle in any of the following situations.







Another special move, involving pawns, is called en passant. This much rarer move may only occur in specific circumstances. Let's say black moves a pawn to b4. If white moves his pawn two spaces to a4, black may then claim en passant, moving to a3 and capturing the white pawn on a4 in the process.



This study will analyze the game of chess through the lens of statistics, probabilities, and combinatorics, which is the study of combinations and permutations of finite sets. The findings will hopefully be used to figure out how to use math to improve chess skills. The goal of this thesis is to ultimately present a way to understand the world's greatest board game via mathematics. Hopefully, the findings will answer the research question: Can math be used to perfect a player's chess skills?

Literature Review

Zermelo's Work Ernst

Zermelo was a German mathematician who received his doctorate from the University of Berlin in 1894 (O'Connor and Robertson). He pioneered research game theory and set theory. In 1913, he published an article titled, "On an Application of Set Theory to the Theory of the Game of Chess." In this article, Zermelo discusses two player games without chance moves. In a 1999 paper, Ulrich Schawlbe and Paul Walker discuss Zermelo's work and contribution to the discussion of chess solvability.

Zermelo presents two questions (Schwalbe and Walker 3). First, when is a player in a "winning" position and can this be defined in a mathematical sense? Second, if a player is in a winning position, can we determine the number of moves needed to force a win? To answer the first question, Zermelo states that there must exist a nonempty set that contains all sequences of moves that produce a victory for one player, say white, no matter how the opponent plays. However, a chess game cannot last forever. This is because a draw can be demanded if an exact position of the pieces is repeated three times. Therefore, this set must be empty. So white would only be able to postpone a loss for a finite amount of moves. This is equivalent to saying that black can force a victory

Now let's focus on the second one, He proved this by contradiction: Assume that white can win in a number of moves greater than the possible positions. Then a winning position would have been repeated at least once. If white had of played the winning move the first time, he would have won in fewer moves than the amount of possible positions.

Countability

For the very first move of the game, white has 20 options to choose from. White can move any of the eight pawns up one or two squares or he can move either knight up and to the left or up and to the right. Similarly, black has 20 moves to choose from for his turn. This leaves us with 400 (because 20×20 = 400) possible variations after just the first two moves.

In 1950, Claude E. Shannon wrote a paper titled "Programming a Computer for Playing Chess." Shannon's purpose of writing this paper was to describe how a computer could be programmed to analyze data and solve problems given by that data. He hoped this problem-solving technology could be used not just in chess, but in many areas such as telephone circuits, music, and language translation.

Shannon claims that it is feasible to play a perfect game or to build a machine to do so. From a given position this machine must consider every possible move, then evaluate all possible moves for the opponent, and so on until the end of the game. Since chess is not an infinite game. The machine would then work backward from the end to see if it can force a win with a particular set of moves (Shannon 4).

Solvability

To better analyze the solvability of chess, we can look at games similar to chess that have been solved or nearly solved.In a 1996 paper, "Solving the Game of Checkers," Jonathan Schaeffer and Robert Lake ask the question, is it possible to program a computer to play checkers perfectly? If so, checkers would be a solved game. The paper states that there are three ways of solving a board game: publicly, practically, and provably. Publically solving checkers means convincing the public that the game is solved. Practically solving means building a program powerful enough to consistently beat the best humans. Provably solving checkers means creating a perfect program that cannot lose.

In their paper, Schaeffer and Lake mainly focus on Chinook, a computer program created by Schaeffer and his team. This checkers playing program was the best in the world at the time. It was, however, not perfect.

In 2007, Schaeffer announced that checkers was finally solved. From the starting position, if two players play a perfect game, making the optimal move on each turn, the result will be a draw. Grandmasters had long assumed that two perfect players would play to a draw.

Since the 1950s, programmers have attempted to build machines that are unbeatable at chess. Garry Kasparov, arguably the greatest chess player of all time, discusses his experiences with chess computers in a 2010 article, “The Chess Master and the Computer.” In 1985, Kasparov played against 32 different computers at once Kasparov won every game with a perfect score of 32-0. This was not a big surprise at the time. Most people expected the grandmaster to easily take down the machines (Kasparov).

In 1996, Kasparov faced off against IBM’s supercomputer Deep Blue. He barely beat the machine. A year later, after IBM doubled Deep Blue’s processing power, Kasparov lost the rematch. In 2003, he played a few matches against two commercially available programs. Both of these matches ended in a draw.

“Chess is far too complex to be definitively solved with any technology we can conceive of today” (Kasprov). Unfortunately, many chess computing projects around the world lost funding and shut down after Kasparov’s defeat in 1997.

Data Analysis

Preliminaries

Combinatorics is essentially the mathematics of counting. It often deals with questions that ask “how many?” (Mazur 1). In this paper, we will be exploring chess and all the different ways a chess game can play out. We will be asking a lot of “how many” questions, so a basic understanding of combinatorics will be necessary.

There are two basic counting principles that are vital to the mathematics of counting. The first of these, the Addition Principle, states:

Assume there are n1 ways for event E1 to occur, n2 ways for event E2 to occur, …, and nk ways for event Ek to occur. If these ways for the different events to occur are pairwise disjoint, then the number of ways for at least one of the events E1, E2, …, or Ek to occur is given by:

$$n_1 + n_2 + \cdots + n_k = \sum_{i=1}^{k} n_k$$

To demonstrate this, let’s imagine two cities that exist a large distance away from each other. We will call the first city, X and the second city, Y. There are four ways that one can drive from X to Y. There are two ways one can fly from X to Y. And there are three ways one can sail from X to Y. So the number of ways that someone can drive, fly, or sail from X to Y is 4 + 2 + 3, which equals 9. In this example, driving from X to Y is E1, flying from X toY is E2, and sailing from X to Y is E3. The Addition Principle might seem somewhat elementary but it is an imperative building block of combinatorics.

The next basic principle, called the Multiplication Principle, states: Assume that an event E can be decomposed into r ordered events E1, E2, …, Er, and that there are n1 ways for the event E1 to occur, n2 ways for the event E2 to occur, …, nr ways for the event Er to occur. Then the total number of ways for the event E to occur is given by:

$$n_1 \times n_2 \times \ldots \times n_r = \prod_{i=1}^{r} n_i$$

To demonstrate this, we will again imagine cities. This time we will have cities W, X, Y, and Z. Our goal is to travel from W to Z. The only way to do this, is by first passing through X and then Y. There are two ways to get from W to X, three ways to get from X to Y, and 5 ways to get from Y to Z. So the total number of ways to get from W to Z is 2×3×5, which equals 30.

The next two vital combinatorics operations are permutations and combinations. A permutation is a way of ordering distinct objects from a given set. Let set A = {a, b, c, d}. Then finding the 3-permutations of A would mean finding all the ways we can arrange three elements from set A. Let n be the number of distinct objects in a set B. Then the way of arranging any r objects of B would be given by:

nPr = n!(n-r)! (where n! = n(n-1)(n-2)...(2)(1))

Combinations differ from permutations in that the order does not matter in a combination. A combination of a set Q is simply a subset of Q. Let Q = {w, x, y, z}. Finding the 3-

combinations of Q would mean finding all the ways we can combine three elements from Q. Let n be the number of distinct objects in a set B. Then the way of combining any r objects of B would be given by:

nCr = n!r!(n-r)!

Probability and Statistics

When observing the rules of chess, one might make some assumptions about the outcome of a game. For instance, one might say that white has a clear advantage since the white player moves first. Another assumption might be that some pieces have more value than others. In his paper, "Evaluation of Material Imbalances," Larry Kaufman assigns a number value to each chess piece. He gives the pawn a value of 1 and rates the other pieces accordingly.

Piece	Value
Pawn	1
Rook	5
Knight	3 ¼
Bishop	3 ¼
Queen	9 ¾

By looking at the table above, we might assume that the queen is the most powerful piece in the game. We can test this assumption by observing a collection of games that have been played. Obviously, a bigger sample size yields a more accurate estimate. It would be very tedious to compile a list of games and analyze them one by one. Fortunately, we can use computer programming to do the job for us.

To test this hypothesis, my mathematics professor and I wrote a program that analyzes chess games, tells which queen was taken first if any, and tells which player won the game. With this information, we were able to test my hypothesis. The details of this program will be explained in the methodology chapter.

For every game in which a queen was captured, there are four different possibilities:

1) The white queen was captured first and the winner was white (w_white)
2) The white queen was captured first and the winner was black (w_black)
3) The black queen was captured first and the winner was white (b_white)

4) The black queen was captured first and the winner was black (b_black)

The programs tallies up the amount of each of the four outcomes and we are left with our results. The data set consisted of 20,058 games. The results are given below:

A. w_white: 1926
B. w_black: 3535
C. b_white: 3822
D. b_black: 1917

This gives us a total of 11,200 games in which a queen was taken. First let us try to figure out if white has a major advantage over black. Out of these 11,200 games, white won 5748. If we take 5,748 divided by the total number of games, we get 5,748/11,200 ≈ 0.5132 = 51.32%. So for this data set, white won just over 51% of the games. From this, we cannot say for sure that white has a clear advantage since black won nearly the same amount of games as white.

Now we will look at how losing a queen affects the chances of winning. Adding together w_white and w_black, we get the amount of games where the white queen was taken first: 5,461. Out of these games, white only won 1,926. So we divide 1,926 by 5,461 to get approximately 0.3527, or 35.27%. As we can see, white's probability of winning drops from 51.32% to 35.27% just from losing the queen.

Now let's look at the results when the black queen is taken first. By adding b_white and b_black, we get 5,739. Out of these games, black won 1,917 times. By dividing 1,917 by 5,739, we get approximately 0.3340 or 33.4%. So black's probability of winning drops from 48.68% to 33.4% from losing the queen.

From our analysis we can deduce that a player's odds of winning are dramatically reduced upon losing a queen. This analysis is fairly simple, but it demonstrates how math can be used to influence chess strategy. Understanding how losing certain pieces affects the chances of winning can certainly help to improve a player's abilities.

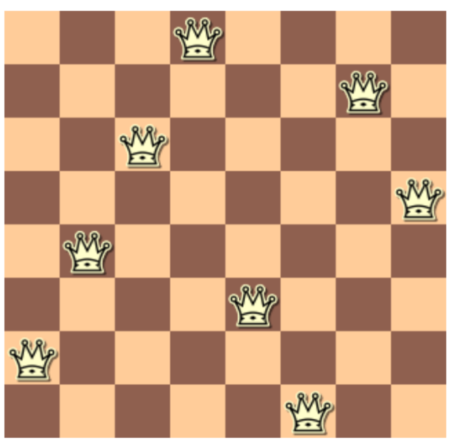
Chess Puzzles

An easy way to observe the workings of math in chess is to look at a few chess puzzles. The most famous of these is the 8-queens puzzle. The problem is stated as follows: can you place eight queens on a chessboard so that no queen is threatening any other? If, Sohow many ways can this be done?

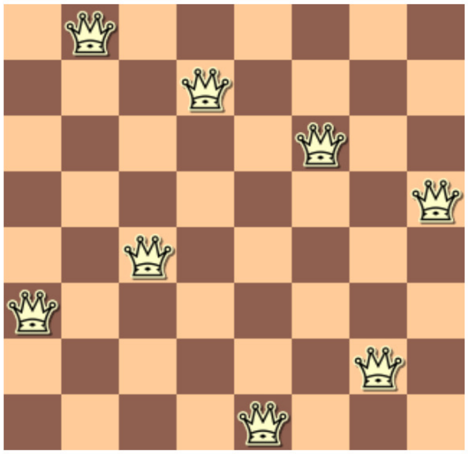
After all, it is highly unlikely that eight queens would exist in a single chess game. Each chess game begins with two queens. For a game to contain eight queens, six pawns must have reached the opposite edge of the board, with no queens being captured. It is true that the event of having eight queens is highly unlikely, but the purpose of the puzzle is to think critically and solve a problem. First we shall note that the maximum number of non-attacking queens that could exist on a chessboard is eight. That is, nine or more queens could not be placed on a chessboard so that none would threaten any other

If we place our queens one at a time, the first queen White 16 will have 64 options. The second queen will then have 63 options and so on. So the number of ways eight queens can be placed on the board would be 64×63×62×61×60×59×58×57 and eliminate all the redundant combinations, we divide by 8!.We note that swapping two queens will produce the same solution. Therefore, the number of unique ways that eight queens can be placed on the board is (64×63×62×61×60×59×58×57)/(8!), which equals 4,426,165,368.

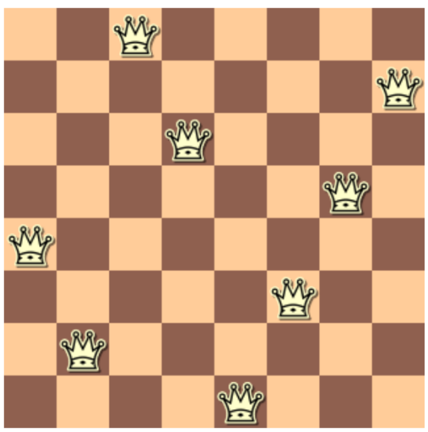
The following board is a solution to the 8-queens puzzle. Each queen has its own row and column. Also, no two queens exist in the same diagonal. We now see that there is at least one solution for the 8-queens puzzle:

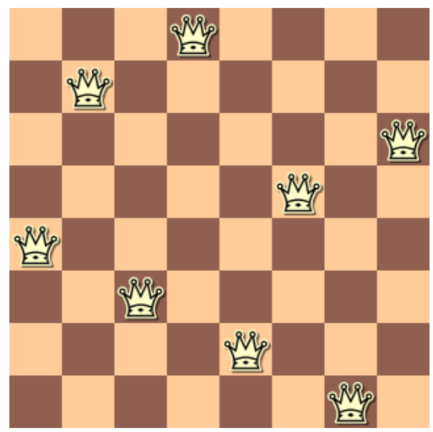


But is this the only solution? On examining this placement, we see that rotating the queens 90 degrees about the center of the board will produce a new solution.

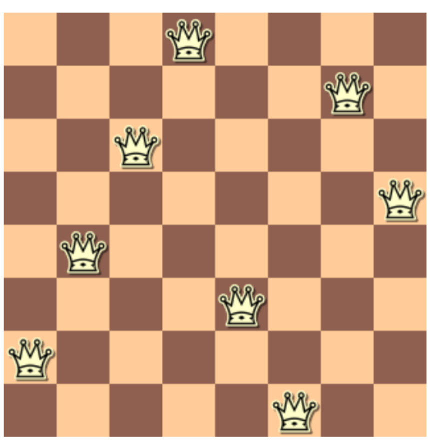


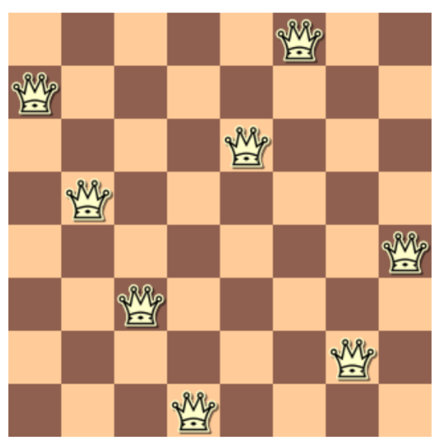
We can then rotate it 90 degrees two more times to get the following two solutions:



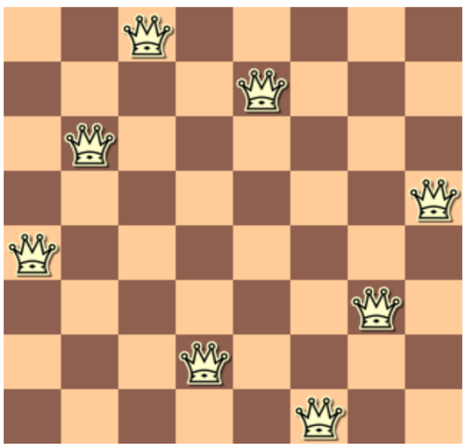


In addition to rotating, we can also reflect each solution across the center line (the imaginary horizontal line between rows 4 and 5). The following solutions are reflections of each other. Reflecting the solution on the left across the center line yields the solution on the right:





As we have seen, each solution can generate other related solutions through rotation and reflection. We will call each group of these related configurations solution families. According to Guti´errez-Naranjo et al., there are twelve families of solutions (199). Every family except for one consists of eight related configurations (four rotations and four reflections). The following board belongs to the solution family that only contains four configurations. This is because the board has rotational symmetry. Rotating the board 180 degrees yields the same configuration.



This leaves us with a total of 92 solutions for the 8-queens puzzle out of the 4,426,165,368 possibilities. This means that 0.000002% of the possible combinations are solutions to the puzzle.

Let us now look at another puzzle of non-attacking pieces. After eight are placed, each row will contain a rook and any future rooks that are placed would be put in an already occupied row. Similar to the queens, there are 4,426,165,368 possible combinations of eight rooks on a chessboard.This formula may be used for n rooks on an n×n board. There will be n! ways to arrange n rooks on an n×n chess board so that they are non-attacking.

Methodology

I hypothesized that being the first player to lose the queen would dramatically reduce the chances of winning. In order to test this hypothesis, my professor and I wrote a program that analyzes a large data set of chess games. We found a data set consisting of 20,058 games at kaggle.com. These games were compiled from lichess.org, a free chess website used by players around the world.

The data set came in the form of a Microsoft Excel spreadsheet. Each game is represented by the moves made and the piece that made each move. The notation “Nf6” means that a knight was moved to square f6. The “+” sign is used to represent a move that puts the opposing king in check. The notations “O-O” and “O-O-O” are used to denote castling. The following example shows what a typical game looks like.

d4 d5 Nf3 Bf5 Nc3 Nf6 Bf4 Ng4 e3 Nc6 Be2 Qd7 O-O O-O-O Nb5 Nb4 Rc1 Nxa2 Ra1 Nb4 Nxa7+ Kb8Nb5 Bxc2 Bxc7+ Kc8 Qd2 Qc6 Na7+ Kd7 Nxc6 bxc6 Bxd8 Kxd8 Qxb4 e5 Qb8+ Ke7 dxe5 Be4 Ra7+ Ke6 Qe8+ Kf5 Qxf7+ Nf6 Nh4+ Kg5 g3 Ng4 Qf4+ Kh5 Qxg4+ Kh6 Qf4+ g5 Qf6+ Bg6 Nxg6 Bg7 Qxg7#

We first store each queen’s starting position. If a queen moves, her new position will be stored. If there is a move in which a piece is captured, an “x” will appear, followed by the square where the capture took place. We then store the winner of that particular game (in this study we simply ignore games where neither queen was captured).

In their 1973 paper, “Skill in Chess,” Herbert Simon and William Chase estimate roughly that the typical grandmaster has spent 10,000 to 50,000 hours staring at chess positions (Simon and Chase 402). A vital component in gaining expertise in chess is deliberate practice. Grandmasters spend hundreds of hours honing a particular skill (Charness et al. 152). Perhaps grandmasters spend a lot of time learning to play without a queen and are therefore better equipped to win without one. This would be worth looking into in future studies.

Conclusion

Chess is not overly complex and yet it takes years to master. It is played by people of all ages from countless different backgrounds. A myriad of different strategies may be employed in the game. At any point in a game, each player knows every move that has taken place up to that point. Each player also has knowledge of every move their opponent can possibly make.

Through probability, statistics, and combinatorics, we have observed possibilities, probabilities, and solutions to chess puzzles. Using combinatorics, we demonstrated the vast amount of possible chess games. We used probabilities and statistics to establish the importance of the queen. We also used math to analyze and solve some chess puzzles. There are just too many possible moves and strategies in chess for the existence of an all-knowing, all-powerful player.

Unfortunately, until chess is provably solved, a player's skills cannot be perfected. While this may be true, chess skills can still be greatly improved upon. As we have shown, math exists inherently in the game. A better understanding of the mathematics of chess will lead to a better understanding of the game itself. This will ultimately lead a player to improve their chess playing abilities.

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