REAL LIFE APPLICATIONS OF LINEAR ALGEBRA UNDER MATRIX MULTIPLICATIONS

Nandhini.V [1], Jeya sowmiya.S [2], Ramya.M [3], Sri Ranjani[4]

[1]Assistant Professor, Department of mathematics, Sri Krishna Arts and Science College,
[2, 3,4]UG Student, Sri Krishna Arts and Science College.

[1]nandhinivelusamy@skasc.ac.in ,
[2] jeyasowmiya19bma012@skasc.ac.in
[3] ramyam19bma021@skasc.ac.in
[4] sriranjanis19bma032@skasc.ac.in

Abstract – In this paper, we will discuss how linear algebra is used in computer graphics worked under matrix multiplication, and we will also look on to the basic concept regarding computer graphics. In a 3D graphics, the process of transforming points and direction vectors from one co-ordinate space to another is used by matrices. The present study is to discuss the concepts of linear algebra that are applied in computer graphics.

Keywords: *linear algebra, matrix multiplication.*

I. INTRODUCTION

Linear algebra is the branch of mathematics concerning linear equations such as $a_1x_1 + a_2x_2 + ... + a_nx_n = b$ and their representations through matrices and vector space. Linear algebra is central to almost all areas of mathematics. For instance, linear algebra is fundamental in modern presentations of geometry, including for defining basic objects such as lines, planes and rotations. Also, functional analysis may be basically viewed as the application of linear algebra to spaces of functions. Linear algebra is also used in most sciences and engineering areas, because it allows modelling many natural phenomena, and efficiently computing with such models. For nonlinear systems, which cannot be modelled with linear algebra, linear algebra is often used as a first-order approximation.

The matrix is used in computer graphics over a large area or number of industries. The matrices are the major mathematical tool which will help to build and develop to manipulate a realistic animation from a figure of polygon. At the earliest stage of video gaming industry to rely more on computer graphics. Hence, the basic necessity to program 3D video game is done by the concept of matrix multiplication.

The motivation of this paper is to provide details about Linear Algebra and some application approach towards in computer graphics, which is worked under matrix multiplication. The rest of the article are organized to be Section II, contains preliminaries, Section III has an approach towards matrix in computer graphics. In the last section, conclusions are given.

II. PRELIMINARIES

- **Linear system:**

A finite set of linear equations in a finite set of variables, for example: x_1, x_2,..., x_n or x, y, z is called system of linear equations, forms a fundamental part of linear algebra. Linear algebra through vector spaces and matrices, many problems may be interpreted in terms of linear systems.

- **Vectors:**

A vector is simply an element of a vector space, period. It has a phenomenon which contains two independent properties: magnitude and direction, such as: weight, force, magnetic field, acceleration, etc.

- **Matrix:**

A matrix is a collection of numbers arranged in a fixed number of rows and columns. Usually the numbers are real.

Example: $\begin{bmatrix} r1 & r2 & r3 \end{bmatrix}$-row matrix, $\begin{bmatrix} c1 \\ c2 \\ c3 \end{bmatrix}$-column matrix

$$\begin{bmatrix} a1 & a2 & a3 \\ a4 & a5 & a6 \\ a7 & a8 & a9 \end{bmatrix}$$

- **Matrix equation:**

A matrix equation is an equation of the form Ax=B, where A is an m×n matrix, B is a vector in R^m and x is a vector whose coefficients x_1, x_2..., x_n are unknown. There are a wide variety of problems that come up in computer graphics that require the numerical solution of matrix equations. Matrix formulations of problems come up often enough in graphics that I rank this area very high on the list of topics to know.

$$x_1 \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n & a_{12} & \cdots & a_{1n} \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}.$$

- **MATRICES AND SCALAR MULTIPLICATION:**

A matrix is a rectangular arrangement of numbers into rows and columns. When we work with matrices, we refer to real numbers as scalars. The term scalar multiplication refers to the product of a real number and a matrix. In scalar multiplication, each entry in the matrix is multiplied by the given scalar.

- **COMPUTER GRAPHICS:**

Computer graphics is the disciple of generating images with the aid of computers. Today, computer graphics is a core technology in digital photography, film, video games, cell phone and many specialized applications. Computer graphics can be a series of image or a single image. It is often abbreviated as CG, or typically in the context of film as CGI.

- **TYPES IN CG:**

There are two kinds of computer graphics:

1) RASTER (composed of pixels)
2) VECTOR (composed of paths)

1) RASTER GRAPHICS:

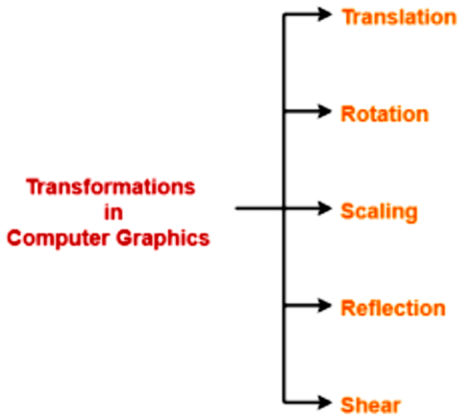
Raster images are more commonly called bitmap images. A bitmap image uses a grid of individual pixels where each pixel can be different colour or shade. Bitmaps are composed of pixels. In computer graphics or bitmap images is a dot matrix data structure that represents a generally rectangular grid of pixels (points of colour), viewable via a monitor, paper, or other display medium. Raster images are stored in image files with varying formats. A raster is technically characterized by the width and height of the image in pixels and by the number of bits per pixel.

2) VECTOR GRAPHICS:

Vector graphics are computer graphics images that are defined in terms of 2D points, which are connected by lines and curves to form polygons and other shapes. Each of these points has a definite position on the x-axis and y-axis of the work plane and determines the direction of the path. Vector graphics are commonly found today in the SVG, EPS, PDF or AI graphic file formats and intrinsically different from the more common raster graphics file formats such JPEG, PNG, APNG, and GIF.

- **Transformation and its types :**

There are 5 main types of transformations:



These transformation falls into two categories:

- Rigid transformation that do not change the shape or size of the pre image.
- Non rigid transformation that change the size but not the shape of the pre image.

1. SCALING:

It is used to alter or change the size of objects. The change is done using scaling factors.

There are two scaling factors, i.e.

- S_x in x direction
- S_y in y direction

If the original position is x and y. Scaling factors are S_x and S_y then the value of coordinates after scaling will be x1 and y1. If the picture to be enlarged to twice its original size then Sx = Sy = 2. If Sx and Sy are not equal then scaling will occur but it will elongate or distort the picture. If scaling factors are less than one, then the size of the object will be reduced. If scaling factors are higher than one, so the size of the object will be enlarged.

SCALING:

If the picture is to be zoomed twice of its original size then the transformation matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is multiplied with the corresponding taken points of the image.

2. TRANSLATION:

Translation is a movement of objects without deformation. Every position or point is translated by the same amount. For translating polygon, each vertex of the polygon is converted to a new position. Similarly, curved objects are translated. To change the position of the circle or ellipse.Its centre coordinates are transformed.

3. ROTATION:

It is a process of changing the angle of the object. Rotation can be clockwise or anticlockwise. For rotation, we have to specify the angle of rotation and rotation point. Rotation point is also called a pivot point. It is print about which object is rotated.

TYPES OF ROTATION:

1) Anticlockwise rotation
2) Clockwise rotation

The positive value of the pivot point (rotation angle) rotates an object in a counter - clockwise (anti-clockwise) direction. The negative value of the pivot point (rotation angle) rotates an object in a clockwise direction. When the object is rotated, then every point of the object is rotated by the same angle. Rotations can also be done in straight lines, polygon, curved lines, circle and ellipse.

4. REFLECTION:

It is a transformation which produces a mirror image of an object. The mirror image can be either about x-axis or y-axis.

TYPES OF REFLECTION:

1. Reflection about the x-axis
2. Reflection about the y-axis
3. Reflection about an axis perpendicular to x-y plane and passing through the origin
4. Reflection about line y=x

5. SHEARING:

It is transformation which changes the shape of object. The sliding of layers of object occur.

The shear can be in one direction or in two directions.

Shearing in the X-direction:

Shearing in the Y-direction:

Shearing in X-Y directions:

6. DILATION:

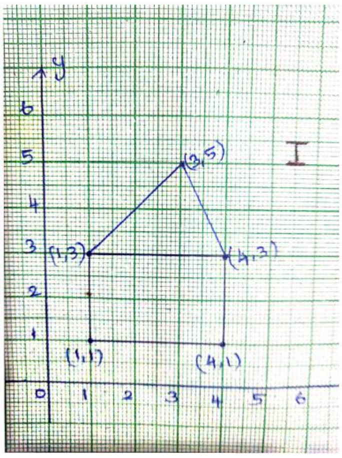
Dilation is usually represented by is one of the basic operations in mathematical Morphology.

The structuring element is moved across every pixel in the original image to give a pixel in a new processed image during morphology.

III. MATRIX IN COMPUTER GRAPHICS

PROBLEM 1:

ALTER THE SIZE OF THE GIVEN IMAGE BY USING SCALING PROCESS



SCALING:

If the picture is to be zoomed twice of its original size then the transformation matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is multiplied with the corresponding taken points of the image.

Solution:

Here we have taken the points of the house as the required matrix now it is to be zoomed

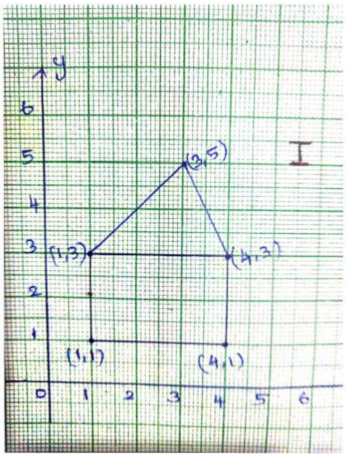
Twice, $\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix}$

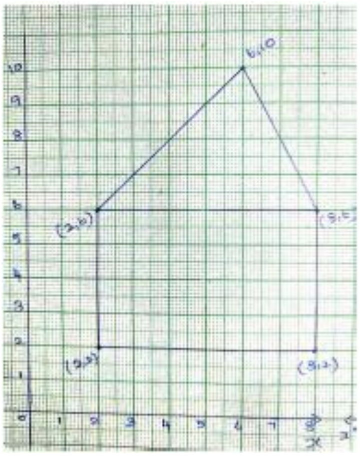
IMAGE AFTER SCALING:

$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 2 & 6 \\ 8 & 6 \\ 2 & 2 \\ 8 & 2 \end{bmatrix}$$

Interpretation:

In a graph, the points of the image after scaling from its original position and so the required image are formed.





BEFORE SCALING AFTER SCALING

ROTATION:

If the picture is to be rotated 90^0 from its original size then the transformation matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is multiplied with the correspondence taken points of the image.

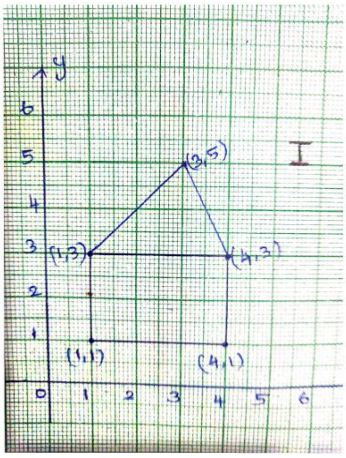
If the picture is to be rotated 180^0 from its original size then the transformation matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ is multiplied with the correspondence taken points of the image.

If the picture is to be rotated 270^0 from its original size then the transformation matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is multiplied with the correspondence taken points of the image.

If the picture is to be rotated 270^0 from its original size then the transformation matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is multiplied with the correspondence taken points of the image.

Problem 2:

ROTATE THE GIVEN HOUSE INTO 90^0 FROM ITS ORIGINAL POSITION.



Solution:

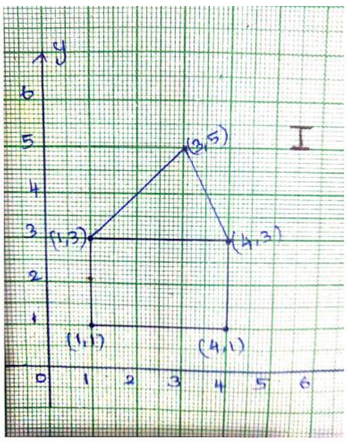
$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix}$$

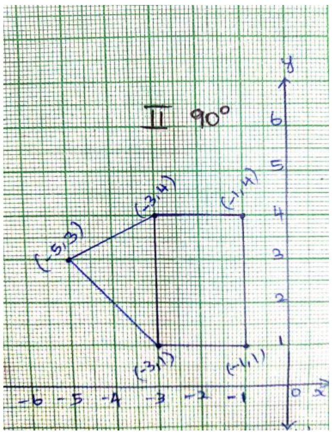
IMAGE AFTER ROTATION THROUGH 90^0

$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ -3 & 1 \\ -3 & 4 \\ -1 & 1 \\ -1 & 4 \end{bmatrix}$$

Interpretation:

In a graph, the points of the image after a rotation through 90^o from its original position and so this lies in the second quadrant.

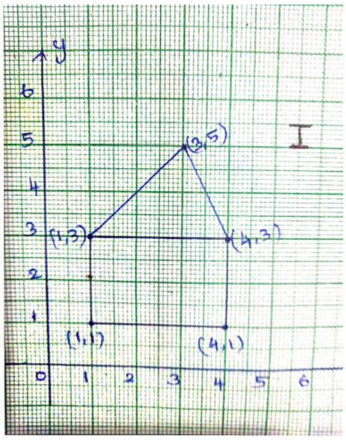




BEFORE ROTATION AFTER ROTATION

Problem 3:

ROTATE THE GIVEN POINTS OF AN IMAGE INTO 180^0 FROM ITS ORIGINAL POSITION.



Solution:

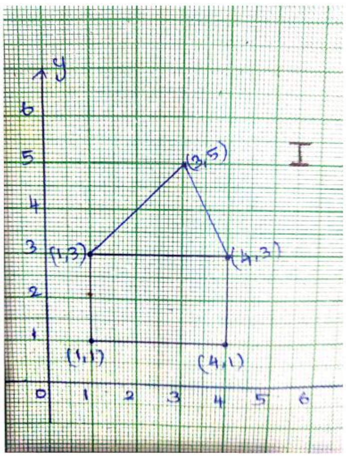
$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix}$$

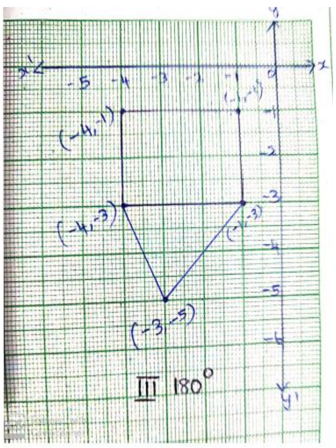
IMAGE AFTER ROTATION THROUGH 180^0:

$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & -5 \\ -1 & -3 \\ -4 & -3 \\ -1 & -1 \\ -4 & -1 \end{bmatrix}$$

Interpretation:

In a graph, the points of the image after a rotation through 180^o from its original position and so this lies in the third quadrant.

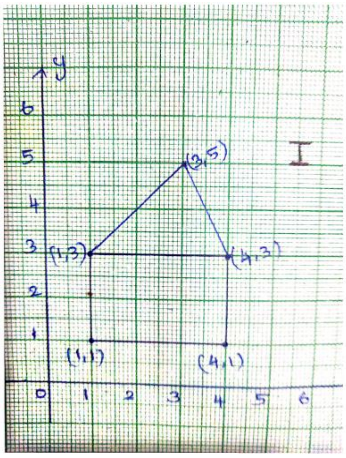




BEFORE ROTATION AFTER ROTATION

Problem 4:

ROTATE THE GIVEN POINTS OF AN IMAGE INTO 270^0 FROM ITS ORIGINAL POSITION



Solution:

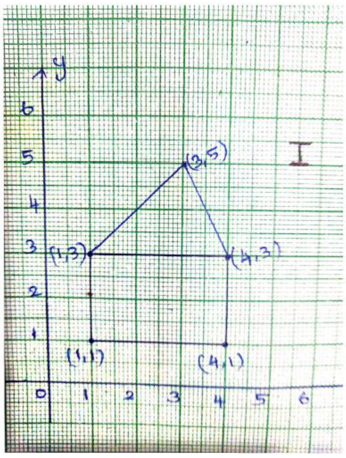
$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix}$$

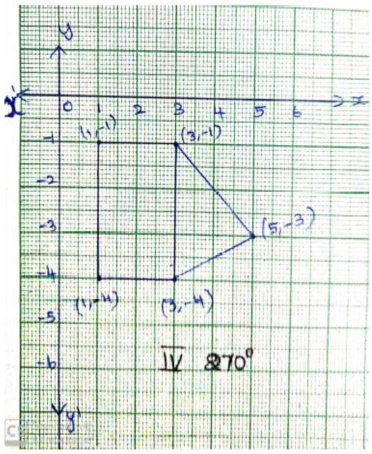
IMAGE AFTER ROTATION THROUGH 270^0:

$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 3 & -1 \\ 3 & -4 \\ 1 & -1 \\ 1 & -4 \end{bmatrix}$$

Interpretation:

In a graph, the points of the image after a rotation through 270° from its original position and so this lies in the fourth quadrant.

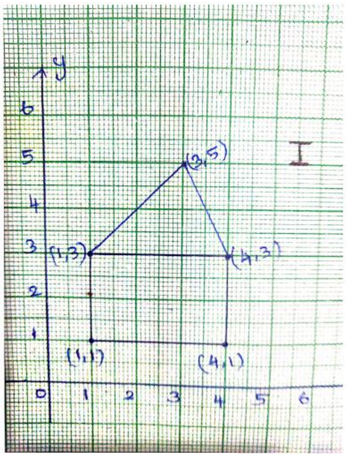




BEFORE ROTATION AFTER ROTATION

Problem 5:

ROTATE THE GIVEN POINTS OF AN IMAGE INTO 360^0 FROM ITS ORIGINAL POSITION.



Solution:

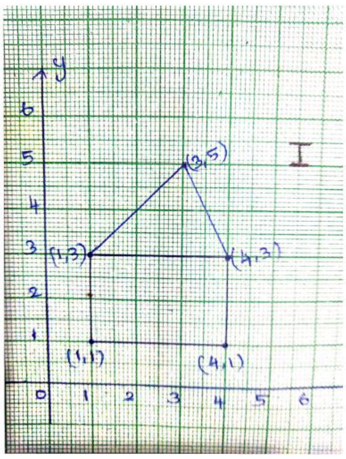
$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix}$$

IMAGE AFTER ROTATION THROUGH 360^0:

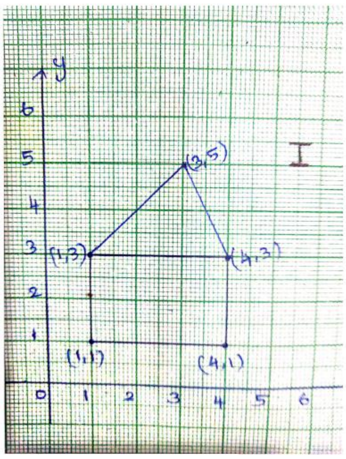
$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 3 \\ 4 & 3 \\ 1 & 1 \\ 4 & 1 \end{bmatrix}$$

Interpretation:

In a graph, the points of the image after a rotation through 360° from its original position and so this lies in the first quadrant.



BEFORE ROTATION



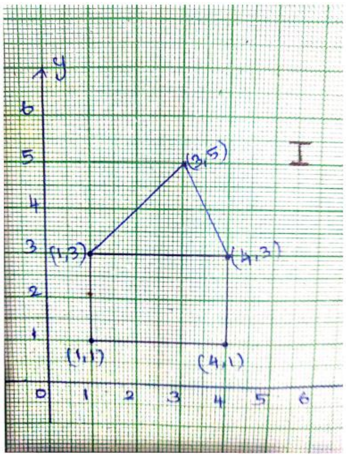
AFTER ROTATION

REFLECTION:

To view the reflection of the taken image it should be multiplied with the transform matrix$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$.

Problem 6:

REFLECT THE IMAGE OF A GIVEN POINTS FROM ITS ORIGINAL POSITION.



Solution:

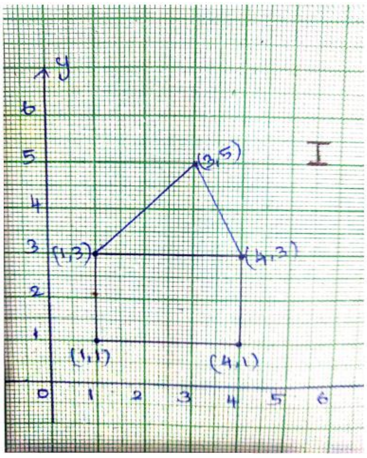
$$\begin{bmatrix}3 & 5\\1 & 3\\4 & 3\\1 & 1\\4 & 1\end{bmatrix}$$

IMAGE AFTER REFLECTION

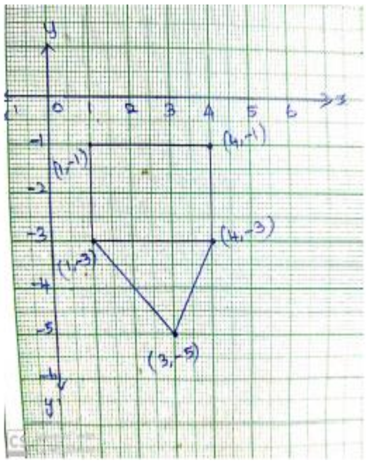
$$\begin{bmatrix}3 & 5\\1 & 3\\4 & 3\\1 & 1\\4 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}=\begin{bmatrix}3 & -5\\1 & -3\\4 & -3\\1 & -1\\4 & -1\end{bmatrix}$$

Interpretation:

In a graph, the points of the image for the reflected position lie in the fourth quadrant.



BEFORE REFLECTION



AFTER REFELECTION

IV. CONCLUSION

In this project, I have given details about Linear algebra and some applications of Linear algebra. An approach of Linear algebra in Computer graphics is presented. Mainstream Applications of computer graphics can be seen in every type of media including animation, movies and videogames. In computer graphics, how matrix is used as an ability to transform geometric data into different coordinate systems. In easy terms, we say that the elements of a matrix that undergoes a transformation. So we try to understand how images are being scaled, translated and rotated on the computer screen using a matrix transformation. Finally, we concluded that many of the computer graphics applications are worked under matrix multiplication in Linear algebra.

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