NEUTROSOPHIC VAGUE IDEAL TOPOLOGY

MOUNIKA.P1,SOWBARNIKAA.A2,SHAMRUTHI.M3

1Assistant Professor, Sri Krishna Arts And Science College, Coimbatore, Tamil Nadu, India.

2Student, Sri Krishna Arts And Science College Coimbatore, Tamil Nadu, India.

3Student,Sri Krishna Arts And Science College Coimbatore, Tamil Nadu, India.

ABSTRACT

In this paper we developed and constructed a new concept named neutrosophic vague ideal topology. By the definitions and the necessary example we have defined a neutrosophic vague ideal topology which is discussed. Also we have introduced the theorems in neutrosophic vague.

KEYWORDS

Neutrosophic vague set, neutrosophic vague ideal, neutrosophic vague topological space,neutrosophic vague topology.

1.INTRODUCTION

Zadeh defined and introduced the fuzzy set in 1965 which deals with the degree of membership. Topology has become one of the powerful instrument of mathematical research. Topology is also the modern version of the geometry. Smarandache has defined the neutrosophic set in 1998.

Smarandache and Abdel-Basset.et.al have proposed various methods for neutrosophic sets. The concept of neutrosophic vague set has been introduced by Shawkat Alkhazalehin the year 2015 as the combination of vague set and neutrosophic set. In this paper we have discussed about the neutrosophic vague ideal topology.

2. DEFINITION

Definition 2.1

(Smarandache (2005)) A neutrosophic set A on the universe of discourse X is defined as A = {< x; $T_A(x)$; $I_A(x)$; $F_A(x)$ >; x ∈ X}, where T; I; F : X →] $^-0$; 1^+[and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2.2

(Alkhazaleh (2015)) A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X can be written as

A_{NV} = {< x; $\hat{T}_{ANV}$ (x); $\hat{I}_{ANV}$ (x); $\hat{F}_{ANV}$ (x) >; x ∈ X}

whose truth-membership, indeterminacy - membership and falsity-membership functions is defined as

$\hat{T}_{ANV}(x) = [T^-, T^+]$, $\hat{I}_{ANV}(x) = [I^-, I^+]$ and $\hat{F}_{ANV}(x) = [F^-, F^+]$, where

1. $T^+ = 1 - F^-$, $F^+ = 1 - T^-$ and 2. $^-0 \leq T^- + I^- + F^- \leq 2^+$.

Definition 2.3

A neutrosophic vague topology (NVT) on X_{NV} is a family τ_{NV} of neutrosophic vague sets (NVS) in X_{NV} satisfying the following axioms:

- $0_{NV}, 1_{NV} \in \tau_{NV}$
- $G_1 \cap G_2 \in \tau_{NV}$ for any $G_1\ G_2 \in \tau_{NV}$,
- $\cup G_i \in \tau_{NV}, \forall \{G_i : i \in J\} \subseteq \tau_{NV}$

In this case the pair (X_{NV}, τ_{NV}) is called a neutrosophic vague topological space (NVTS).

3.NEUTROSOPHIC VAGUE IDEAL TOPOLOGY

3.1.Definition

Let W_{NV} is non-empty set and L_{NV} a nonempty family of NV_S. We will call L_{NV} is a neutrosophic vague ideal (NVL for short) on if

- $0_{NV} \in L_{NV}$
- $A_{NV1} \in L_{NV}$ and $A_{NV2} \subseteq A_{NV1} \Longrightarrow A_{NV2} \in L_{NV}$ (heredity)
- $A_{NV1} \in L_{NV}$ and $A_{NV2} \in L_{NV} \Longrightarrow A_{NV1} \cup A_{NV2} \in L_{NV}$ (finite additivity)

The smallest and largest neutrosophic ideals on a non-empty set W_{NV} are 0_{NV} and NVs on W_{NV} . Also, NV. L_{NVf} ,NV. L_{NVc} are denoting the neutrosophic vague ideals (NVL for short) of neutrosophic vague subsets having finite and countable support of W_{NV} respectively. Moreover, if A_{NV1} is a nonempty NVs in W_{NV} , then A2 ⊆ NVs : $A_{NV1} \subseteq A_{NV2}$ is an NVL on W_{NV} . This is called the principal NVL of all NVs of denoted by NVL (A1) .

Remark

- If $1_{NV} \notin L_{NV}$, then L_{NV} is called neutrosophic vague proper ideal.
- If $1_{NV} \in L_{NV}$,then L_{NV} is called Neutrosophic vague improve ideal.
- $0_{NV} \in L_{NV}$.

Example

Let $W_{NV} = \{w_1, w_2, w_3\}$

$A_{NV1} = \{ \frac{w_1}{(0.3,0.2),(0.4,0.2),(0.5,0.6)}, \frac{w_2}{(0.1,0.5),(0.7,0.2),(0.9,0.2)}, \frac{w_3}{(0.5,0.6),(0.4,0.6),(0.3,0.1)}\}$,

$A_{NV2} = \{ \frac{w_1}{(0.1,0.8),(0.5,0.3),(0.8,0.2)}, \frac{w_2}{(0.4,0.3),(0.5,0.6),(0.8,0.4)}, \frac{w_3}{(0.8,0.2),(0.3,0.5),(0.4,0.9)}\}$ and

$A_{NV3} = \{ \frac{w_1}{(0.6,0.3),(0.5,0.4),(0.3,0.4)}, \frac{w_2}{(0.7,0.8),(0.4,0.3),(0.4,0.6)}, \frac{w_3}{(0.9,0.4),(0.6,0.7),(0.6,0.5)}\}$, then the family $A_{NV4} = 0_{NV}$, A_{NV1} , A_{NV2} , A_{NV3} of NVs is an NVL on W. Example 2.1.2 Let $W_{NV} = \{w_1, w_2, w_3, w_4, w_5\}$ and A_{NV1} = < w, T_{NV} , I_{NV}, F_{NV}>

Example 3.2

Let $W_{NV} = \{w_1, w_2, w_3, w_4, w_5\}$ and A_{NV1} = < w, T_{NV} , I_{NV} , F_{NV}>

W_{NV}	T_{NV}	I_{NV}	F_{NV}
W_1	(0.6,0.4)	(0.4,0.6)	(0.3,0.7)
W_2	(0.5,0.4)	(0.3,0.5)	(0.4,0.5)
W_3	(0.6,0.5)	(0.8,0.9)	(0.8,0.7)
W_4	(0.3,0.8)	(0.4,0.8)	(0.4,0.7)
W_5	(0.4,0.5)	(0.8,0.5)	(0.6,0.2)

Then the family $L_{NV} = 0_{NV}$, A_{NV1} is an NVL on W.

Remark

The neutrosophic vague ideal by the single neutrosophic vague set 0_{NV} is the smallest element of the ordered set of all neutrosophic vague ideals on W_{NV} .

Definition

Let (W_{NV} , τ_{NV}) be a neutrosophic vague topological spaces (NVTS for short) and L_{NV} be neutrsophic vague ideal (NVL, for short) on W_{NV}. Let A_{NV1} be any NVS of W_{NV} . Then the neutrosophic vague local function NV A^{*NV} (L_{NV}, τ_{NV}) of A_{NV1} is the union of all neutrosophic vague points(NVP, for short) , C_{NV} (α, β, γ) such that if U ∈NV C_{NV} (α, β, γ) and NV A^{*NV} (L_{NV} , τ_{NV}) = ∨ NV C_{NV} (α, β, γ) ∈ W_{NV}: $A_{NV1} \cap U \notin L_{NV}$ for every U neighborhood of C_{NV} (α, β, γ), NV A^{*NV} (L_{NV}, τ_{NV}) is called a neutrosophic vague local function of A_{NV1} with respect to τ_{NV} and L_{NV} which it will be denoted by NV A^{*NV} (L_{NV} , τ_{NV}) , or simply NV $A^{*NV}L$

Example

Let (U_{NV} , τ_{NV}) be the neturosophic vague topological spaces with ideal L_{NV} on U_{NV} and for every $W_{NV} \subseteq U_{NV}$

If $L_{NV} = \{0_{NV}\}$, then NV A^{*NV} (L_{NV} , τ_{NV})=NVcl(A_{NV1}), for any neutrosophic vague set $A_{NV1} \in$ NVs on W_{NV}.

If L_{NV} = all NVs on W_{NV} then NV A^{*NV} (L_{NV} , τ_{NV}) = 0_{NV}, for any $A_{NV1} \in$ NVs on W_{NV}.

Theorem 3.1.1

Let (W_{NV} , τ_{NV} , L_{NV}) be a space and A_{NV1} , $B_{NV1} \subseteq W_{NV}$ then the following statements hold.

(i) If $A_{NV1} \subseteq B_{NV1}$ then $A^{*NV}_{NV_1}$ (L_{NV}) ⊆ $B^{*NV}_{NV_1}$(L_{NV})

(ii) $A^{*NV}_{NV_1}$ (L_{NV}) = NVcl($A^{*NV}_{NV_1}$(L_{NV})) ⊆ NV cl(A_{NV1})

(iii) ($A_{NV1} \cup B_{NV1}$) ^{*NV} (L_{NV}) = $A^{*NV}_{NV_1}$ (L_{NV}) ∪ $B^{*NV}_{NV_1}$ (L_{NV})

(iv)($A_{NV1} \cap B_{NV1}$) ^{*NV} (L_{NV}) = A $^{*NV}_{NV_1}$ (L_{NV}) ∩ B $^{*NV}_{NV_1}$ (L_{NV})

(v) A $^{*NV}_{NV_1}$ (L_{NV}) − B $^{*NV}_{NV_1}$ (L_{NV}) ⊆ (A_{NV1} (L_{NV}) – B_{NV1}) ^{*NV} (L_{NV})

(vi) (A $^{*NV}_{NV_1}$) (L_{NV}) ⊆ (A_{NV1}) ^{*NV} (L_{NV})

(vii)If V_{NV}∈ L_{NV} then (A_{NV1} ∪V_{NV})^{*NV} (L_{NV}) = A $^{*NV}_{NV_1}$ (L_{NV}) = (A_{NV1}−V_{NV}) ^{*NV} (L_{NV})

(viii)If X_{NV}∈τ_{NV} then X_{NV}∩A$^{*NV}_{NV_1}$ = X_{NV} ∩(X_{NV} ∩A_{NV1})^{*NV}⊆ (X_{NV}∩A_{NV1}) ^{*NV} (L_{NV})

Proof:

(i) Let A_{NV1}⊆ B_{NV1}.

A $^{*NV}_{NV_1}$ (L_{NV}) = {w ∈ W_{NV} /U ∩ A_{NV1}∉ L_{NV} , for every U ∈ NV C_{NV} (α, β, γ)}

w ∈ A $^{*NV}_{NV_1}$ (L_{NV}) =⇒ U∩ A_{NV1}∉ L_{NV} , for every U NV C_{NV} (α, β, γ)

=⇒ U ∩B_{NV1}∉ L_{NV} , for every U ∈ NV C_{NV} (α, β, γ)

This shows that w ∈ B$^{*NV}_{NV_1}$. Hence A $^{*NV}_{NV_1}$ (L_{NV}) ⊆ B $^{*NV}_{NV_1}$ (L_{NV})

(ii) To prove A $^{*NV}_{NV_1}$ (L_{NV}) = NV cl(A $^{*NV}_{NV_1}$ (L_{NV})) ⊆ NV cl(A_{NV1})

NV cl A$^{*NV}_{NV_1}$ (L_{NV}) ⊇ A $^{*NV}_{NV_1}$.

Therefore A $^{*NV}_{NV_1}$⊆ NV cl A$^{*NV}_{NV_1}$ (L_{NV})

To Prove A $^{*NV}_{NV_1}$⊇ NV cl(A $^{*NV}_{NV_1}$).

Let w ∈ NV cl(A$^{*NV}_{NV_1}$).

Therefore every U ∈ NV C_{NV}(α, β, γ) intersects A $^{*NV}_{NV_1}$.

A $^{*NV}_{NV_1}$⊆ NV cl(A_{NV1})

Let w ∈ A $^{*NV}_{NV_1}$

=⇒ U ∩A_{NV1} 6= 0_{NV} for every U ∈ NV C_{NV} (α, β, γ)

This shows that w ∈ NV cl(A_{NV1}). Thus A $^{*NV}_{NV_1}$⊆ NV cl(A_{NV1})

Hence A $^{*NV}_{NV_1}$ (L_{NV}) = NV cl A$^{*NV}_{NV_1}$ (L_{NV}) ⊆ NV cl(A$^{*NV}_{NV_1}$).

(iii) ($A_{NV1} \cup B_{NV1}$) ∗NV (L_{NV}) = {w ∈ W_{NV} /U ∩ ($A_{NV1} \cup B_{NV1}$) 6= L_{NV} for every U ∈ NV C_{NV} (α, β, γ)}

= {w ∈ W_{NV} /(U ∩ A_{NV1}) ∪ (U ∩ B_{NV1}) 6

= L_{NV} for every U ∈ NV C_{NV} (α, β, γ)}

= {w ∈ W_{NV} /U∩A_{NV1}∈/ L_{NV} orU∩B_{NV1}) ∈/ L_{NV} for every U ∈ NV C_{NV} (α, β, γ)}

= {w ∈ W_{NV} /U ∩ A_{NV1}∈/ L_{NV} for every U ∈ NV C_{NV} (α, β, γ)}

or {w ∈ W_{NV} /U ∩ B_{NV1}∈/ L_{NV} for every U ∈ NV C_{NV} (α, β, γ)}

= A $^{*NV}_{NV_1}$ (L_{NV}) ∪ B $^{*NV}_{NV_1}$ (L_{NV}).

(iv) By(ii) we know that $A_{NV1} \subseteq B_{NV1}$

=⇒ A $^{*NV}_{NV_1}$ ⊆ B $^{*NV}_{NV_1}$ =⇒ ($A_{NV1} \cap B_{NV1}$) ∗NV ⊆ A $^{*NV}_{NV_1}$ and ($A_{NV1} \cap B_{NV1}$) ∗NV ⊆ B $^{*NV}_{NV_1}$

=⇒ ($A_{NV1} \cap B_{NV1}$) ∗NV (L_{NV}) ⊆ A $^{*NV}_{NV_1}$ (L_{NV}) ∩ B $^{*NV}_{NV_1}$ (L_{NV}).

(v) A $^{*NV}_{NV_1}$ (L_{NV})−B $^{*NV}_{NV_1}$ (L_{NV}) = {w ∈ W_{NV} /U∩A_{NV1}∉ L_{NV} for every U ∈ NV C_{NV} (α, β, γ)} − {w ∈ W_{NV} /U ∩ B_{NV1}∉ L_{NV} for every U ∈ NV C_{NV} (α, β, γ)}

= {w ∈ W_{NV} /U∩(A_{NV1}−B_{NV1}) ∉ L_{NV} for every U ∈ NV C_{NV} (α, β, γ)}

= (A− B) ∗NV (L_{NV})

(vi) By(ii) A $^{*NV}_{NV_1}$ ⊆ NV clA_{NV1} (A $^{*NV}_{NV_1}$) ∗NV (L_{NV})

= NV cl((A $^{*NV}_{NV_1}$) ∗NV (L_{NV})) ⊆ NV cl(A $^{*NV}_{NV_1}$)

To prove NV cl(A^{*NV}) $^{*NV}_{NV_1}$ (L_{NV}) ⊇ (A $^{*NV}_{NV_1}$) ∗NV .

Therefore (A $^{*NV}_{NV_1}$) ∗NV ⊆ NV cl(A $^{*NV}_{NV_1}$) ∗NV (L_{NV})

To Prove (A $^{*NV}_{NV_1}$) ∗NV ⊇ NV cl(A $^{*NV}_{NV_1}$) ∗NV .

Let w ∈ NV cl(A $^{*NV}_{NV_1}$) ∗NV .

Therefore every U ∈ NV C_{NV}(α, β, γ) intersects (A $^{*NV}_{NV_1}$) ∗NV .

Let w ∈ (A $^{*NV}_{NV_1}$) ∗NV =⇒ U ∩A $^{*NV}_{NV_1}$ 6= 0_{NV} for every U ∈ NV C_{NV} (α, β, γ)

This shows that w ∈ NV cl(A_{NV1}) ∗NV .

Thus (A $^{*NV}_{NV_1}$) ∗NV ⊆ NV cl(A_{NV1}) ∗NV

Hence (A $^{*NV}_{NV_1}$) ∗NV (L_{NV}) ⊆ NV cl(A $^{*NV}_{NV_1}$) ∗NV (L_{NV}) = A_{NV1} .

(vii) Let $V_{NV} \in L_{NV}$.

$(A_{NV1} \cup V_{NV})^{*NV} = (A_{NV1})*NV \cup (V_{NV})*NV = (A_{NV1})*NV$

$(A_{NV1} - V_{NV})^{*NV} = \{w \in W_{NV} / U \cap (A_{NV1} - B_{NV1}) \notin L_{NV}$ for every $U \in NV\ C_{NV}(\alpha, \beta, \gamma)\}$

$= \{w \in W_{NV} / U \cap A_{NV1} \notin L_{NV}$ for every $U \in NV\ C_{NV}(\alpha, \beta, \gamma)\}$

$= A^{*NV}_{NV_1}$

Therefore $(A_{NV1} \cup V_{NV})^{*NV}(L_{NV}) = A^{*NV}_{NV_1}(L_{NV}) = (A_{NV1} - V_{NV})^{*NV}(L_{NV})$.

(viii) Let $X_{NV} \in \tau_{NV}$

$X_{NV} \cap A^{*NV}_{NV_1} = X_{NV} \cap \{w \in W_{NV} / X_{NV} \cap A_{NV1} \notin L_{NV}$ for every $U \in NV\ C_{NV}(\alpha, \beta, \gamma)\}$

$= X_{NV} \cap \{w \in W_{NV} / U \cap (X_{NV} \cap A_{NV1}) \notin L_{NV}$ for every $U \in NV\ C_{NV}(\alpha, \beta, \gamma)\}$

$= X_{NV} \cap (X_{NV} \cap A_{NV1})^{*NV}$

$\subseteq (U \cap A_{NV1})^{*NV}$

Hence $X_{NV} \cap A^{*NV}_{NV_1}(L_{NV}) = X_{NV} \cap (X_{NV} \cap A_{NV1})^{*NV} \subseteq (X_{NV} \cap A_{NV1})^{*NV}(L_{NV})$

If $(W_{NV}, \tau_{NV}, L_{NV})$ is a space and $A_{NV1} \subseteq W_{NV}$ then the following hold:

(1) $(A^{*NV}_{NV_1})^{*NV} = A^{*NV}_{NV_1} \subseteq A^{*NV}_{NV_1} = NVcl\ A_{NV1} \subseteq NVcl\ A_{NV1}$.

(2) $(A^{*NV}_{NV_1})^{*NV} \subseteq A^{*NV}_{NV_1} = NVcl\ A_{NV1} \subseteq NVcl\ A_{NV1}$

(3) $NVcl\ (A_{NV})^{*NV} = cl(A^{*NV}_{NV_1})$

(4) $A^{*NV}_{NV_1} \subseteq NV\ cl(A^{*NV}_{NV_1})$

Theorem 3.1.2

Let (W_{NV}, τ_{NV}) be a space and L_{NV1}, L_{NV2} are two neutrosophic vague ideals on W_{NV} and let A_{NV1} be a subset of W_{NV} then $A^{*NV}_{NV_1}(L_{NV1}) \subseteq A^{*NV}_{NV_1}(L_{NV2})$, if $L_{NV1} \subseteq L_{NV2}$.

Proof:

$A^{*NV}_{NV_1}(L_{NV1})$

$= \{w \in W_{NV} / U \cap A_{NV1} \notin L_{NV1}$, for every $U \in NV\ C_{NV}(\alpha, \beta, \gamma)\}$

$\subseteq \{w \in W_{NV} / U \cap A_{NV1} \notin L_{NV2}$, for every $U \in NV\ C_{NV}(\alpha, \beta, \gamma)\}$

$= A^{*NV}_{NV_1}(L_{NV2})$

Hence $A^{*NV}_{NV_1}(L_{NV1}) \subseteq A^{*NV}_{NV_1}(L_{NV2})$

Theorem 3.1.3

If L_{NV1} and L_{NV2} are two ideals on (W_{NV}, τ_{NV}) such that $L_{NV1} \subseteq L_{NV2}$ then

τ^{*NV}_{NV} (L_{NV1}) $\subseteq \tau^{*NV}_{NV}$ (L_{NV2})

Proof:

Let $A_{NV1} \in \tau^{*NV}_{NV}$ (L^{*NV}_{NV})

$\Longrightarrow$ ($W_{NV} - A_{NV1}$) is τ^{*NV}_{NV} (L_{NV1}) closed.

$\Longrightarrow$ NV Cl^{*NV} ($W_{NV} - A_{NV1}$) = ($W_{NV} - A_{NV1}$)

$\Longrightarrow$ ($W_{NV} - A_{NV1}$) ∪ $(W_{NV} - A_{NV1})^{*NV}$ = (WNV - ANV1)

$\Longrightarrow$ $(W_{NV} - A_{NV1})^{*NV}$ (L_{NV1}) ⊆ ($W_{NV} - A_{NV1}$)

$\Longrightarrow$ $(W_{NV} - A_{NV1})^{*NV}$ (L_{NV2}) ⊆ ($W_{NV} - A_{NV1}$)

$\Longrightarrow$ $A_{NV1} \in \tau^{*NV}_{NV}$ (L_{NV2})

Hence τ^{*NV}_{NV} (L_{NV1}) $\subseteq \tau^{*NV}_{NV}$ (L_{NV1}).

Theorem 3.1.4

Let (W_{NV} , τ_{NV} , L_{NV})is a space and $A_{NV1} \subseteq W_{NV}$ then

$A^{*NV}_{NV_1} - (A^{*NV}_{NV_1})^{*NV} \subseteq (A_{NV1} - A^{*NV}_{NV_1})^{*NV}$

Proof:

Let w ∈ $A^{*NV}_{NV_1}$ - $(A^{*NV}_{NV_1})^{*NV}$.

Then $W_{NV} \in A^{*NV}_{NV_1}$.

That is U ∩ $A^{*NV}_{NV_1}$ ∉ L_{NV1} , U ∈ NV C_{NV} (α, β, γ)} .

Thus U ∩ ((A_{NV1} − $A^{*NV}_{NV_1}$) *NV) ∉ L_{NV1} , U ∈ NV C_{NV} (α, β, γ)}

Therefore w ∈ $(A_{NV1} - A^{*NV}_{NV_1})^{*NV}$.

Hence $A^{*NV}_{NV_1}$ − $(A^{*NV}_{NV_1})^{*NV} \subseteq (A_{NV1} A^{*NV}_{NV_1})^{*NV}$.

Theorem 3.1.5

Let (W_{NV} , τ_{NV}) be a space and L_{NV1} , L_{NV2} are two ideals on W_{NV} and let A_{NV1} be a subset of WNV then

$A^{*NV}_{NV_1}$ ($L_{NV1} \cap L_{NV2}$) = $A^{*NV}_{NV_1}$ (L_{NV1}) ∪ $A^{*NV}_{NV_2}$ (L_{NV2}) where $L_{NV1} \cap L_{NV2}$ is an ideal on W_{NV}.

Proof:

Let (W_{NV}, τ_{NV}) be a space and L_{NV1} , L_{NV2} are two ideals on W_{NV} and

let A_{NV1} be a subset of W_{NV}.

Also $L_{NV1} \cap L_{NV2}$ is an ideal on W_{NV}.

$A^{*NV}_{NV_1}$ ($L_{NV1} \cap L_{NV2}$)= {w ∈ W_{NV} /U∩A_{NV1}∉ $L_{NV1} \cap L_{NV2}$,for every U ∈ NV C_{NV} (α, β,γ)}

= {w ∈ W_{NV} /U∩A_{NV1}∉ L_{NV1} or A_{NV1}∉ L_{NV2} ,for every U ∈NVC_{NV} (α, β, γ)}

= {w ∈ W_{NV} /U ∩ A_{NV1}∉ L_{NV1} ,for every U ∈ NV C_{NV} (α, β, γ)} ∪ w ∈ W_{NV} – U ∩ A_{NV1}∉ L_{NV2} for every U ∈ NV C_{NV} (α, β, γ)}

= $A^{*NV}_{NV_1}$ (L_{NV1}) ∪ $A^{*NV}_{NV_1}$ (L_{NV2}).

Therefore, $A^{*NV}_{NV_1}$ (L_{NV1} ∩ L_{NV2}) = $A^{*NV}_{NV_1}$ (L_{NV1}) ∪ $A^{*NV}_{NV_1}$ (L_{NV2}).

Theorem 3.1.6

If (W_{NV} , τ_{NV} , L_{NV}) is a NVT with an ideal L_{NV} and A_{NV1}⊆ $A^{*NV}_{NV_1}$, then $A^{*NV}_{NV_1}$ = NVcl($A^{*NV}_{NV_1}$) = NVcl(A_{NV1}).

Proof:

For every subset A_{NV1} of U,we have

$A^{*NV}_{NV_1}$ = NVcl($A^{*NV}_{NV_1}$) ⊆ NV cl($A^{*NV}_{NV_1}$),by Theorem 3.1.1. (ii)

$A^{*NV}_{NV_1}$ (L_{NV}) = NV cl($A^{*NV}_{NV_1}$ (L_{NV})) ⊆ NV cl(A_{NV1}),

So, A_{NV1}⊆ $A^{*NV}_{NV_1}$

=⇒ NVcl(A_{NV1}) ⊆ NVcl($A^{*NV}_{NV_1}$) and

So, $A^{*NV}_{NV_1}$ = NVcl($A^{*NV}_{NV_1}$) = NVcl(A_{NV1}).

Definition 3.3 Let (W_{NV} , τ_{NV}) be a NVT with an ideal L_{NV} on W_{NV} . The set operator NVcl ^{*NV} is called a neutrosophic vague ∗− closure and is defined as NVcl $^{*NV(ANV1)}$ = A_{NV1}∪ $A^{*NV}_{NV_1}$ for A_{NV1}⊆ a

Theorem 3.1.7

The set operator NVcl ∗NV satises the following conditions:

(i) A_{NV1}⊆ NV cl^{*NV}(A_{NV1}),

(ii) NVcl ^{*NV} (0_{NV}) = 0_{NV} and NV cl^{*NV} (1_{NV}) = 1_{NV} ,

(iii) If A_{NV1}⊂ B_{NV1} , then NVcl ^{*NV} (A_{NV1}) ⊆ NV cl^{*NV} (B_{NV1}),

(iv) NVcl ^{*NV} (A_{NV1}) ∪ NV cl^{*NV} (B_{NV1}) = NVcl ^{*NV} (A_{NV1}∪ B_{NV1}).

(v) NVcl ^{*NV} (NV cl^{*NV} (A_{NV1})) = NVcl ^{*NV} (A_{NV1}).

Proof:

The proofs are clear from Theorem 3.1.1 and the definition 2.1.3.

Note 3.1.1

τ^{*NV}_{NV} (L_{NV} , τ_{NV}) = {$X_{NV} \subset W_{NV}$: NV cl^{*NV} ($W_{NV} - X_{NV}$) = $W_{NV} - X_{NV}$ } . τ^{*NV}_{NV} (L_{NV} , τ_{NV}) is called neutrosophic Vague ^{*NV}-topology which is ner than τ_{NV} (we simply write τ^{*NV}_{NV} for τ^{*NV}_{NV} (L_{NV} , τ_{NV})). The elements of τ^{*NV}_{NV} (L_{NV} , τ_{NV}) are called neutrosophic vague ^{*NV}– open(briey,NV ^{*NV}– open)and the complement of an NV ^{*NV}– open set is called neutrosophic vague ^{*NV}– closed (briey, NV ^{*NV} -closed). Here NVcl ^{*NV} (A_{NV1}) and NVint ^{*NV} (A_{NV1}) will denote the closure and interior of A_{NV1} respectively in (W_{NV} , τ^{*NV}_{NV}).

Remark 3.3

(i) We know from Example 2.1.1 that if L_{NV} = {0_{NV} } then A_{NV1} = NVcl(A_{NV1}). In this case,NVcl ^{*NV} (A_{NV1}) = NV cl(A_{NV1}).

(ii) If (W_{NV} , τ_{NV} , L_{NV}) is a NVL with L_{NV} = {0_{NV}} , then τ^{*NV}_{NV} = τ_{NV} .

Definition 3.4

A basis β(L_{NV} , τ_{NV}) for τ^{*NV}_{NV} can be described as follows: β(L_{NV} , τ_{NV}) = {$A_{NV1} - B_{NV2}$: $A_{NV1} \in \tau_{NV}$, $B_{NV1} \in L_{NV}$ } .

Theorem 3.1.8

Let (W_{NV} , τ_{NV})) be a NVT and L_{NV} be an ideal on W_{NV} . Then β(L_{NV} , τ_{NV}) is a basis for τ^{*NV}_{NV} .

Proof:

We have to show that for a given space (W_{NV} , τ_{NV}) be a NVT and an ideal L_{NV} on W_{NV} ,

β(L_{NV} , τ_{NV}) is a basis for τ^{*NV}_{NV} .

If β(L_{NV} , τ_{NV}) is itself a neutrosophic vague topology,

then we have β(L_{NV} , τ_{NV}) = τ^{*NV}_{NV} and

All the open sets of τ^{*NV}_{NV} are of simple form $A_{NV1} - B_{NV1}$

where $A_{NV1} \in \tau_{NV}$ and $B_{NV1} \in L_{NV}$.

Hence, β(L_{NV} , τ_{NV}) is a basis for τ^{*NV}_{NV} .

Theorem 3.1.9

Let (W_{NV} , τ_{NV} , L_{NV}) be a NVT with an ideal L_{NV} on W_{NV} and A_{NV1} W_{NV}.

If$A_{NV1} \subseteq A^{*NV}_{NV}$, then

(i) $NVcl(A_{NV1}) = NVcl^{*NV}(A_{NV1})$,

(ii) $NVint(W_{NV} A_{NV1}) = NV\ int^{*NV}(W_{NV} A_{NV1})$.

Proof:

(i) Follows immediately from Theorem (2.1.6).

(ii) If $A_{NV1} \subseteq A^{*NV}_{NV_1}$,

then $NVcl(A_{NV1}) = NVcl^{*NV}(A_{NV1})$ by (i) and

so $W_{NV}\ NV\ cl(A_{NV1}) = W_{NV}\ NV\ cl^{*NV}(A_{NV1})$.

Therefore, $NVint(W_{NV} A_{NV1}) = NVint^{*NV}(W_{NV} A_{NV1})$.

CONCLUSION:

This paper consists of the neutrosophic vague ideal topology with some examples and theorems. It may extended to the decomposition in neutrosophic vague ideal topology.

REFERENCE:

1. Chang, C.L. Fuzzy topological spaces, J Math.Anal.Appl. 1968; 24, pp. 182–190.
2. A, Mary Margaret; Trinita Pricilla M; and Shawkat Alkhazaleh. "Neutrosophic Vague Topological Spaces." Neutrosophic Sets and Systems 28, 1 (2019). https://digitalrepository.unm.edu/nss_journal/vol28/iss1/ 14
3. Shawkat Alkhazaleh. Neutrosophic vague set theory. Critical Review. 2015; 10, pp. 29–39.
4. Al-Quran, A.; Hassan, N. Neutrosophic vague soft expert set theory. Journal of Intelligent and Fuzzy Systems, 2016; 30(6), pp. 3691-3702
5. Al-Quran, A.; Hassan, N. Neutrosophic vague soft set and its applications. Malaysian Journal of Mathematical Sciences , 2017; 11(2), pp. 141-163.
6. Al-Quran, A.; Hassan, N. Neutrosophic vague soft multiset for decision under uncertainty. Songklanakarin Journal of Science and Technology, 2018; 40(2), pp. 290-305.
7. Nabeeh, N. A.; Smarandache, F.; Abdel-Basset, M.; El-Ghareeb, H. A.; Aboelfetouh, A. An Integrated Neutrosophic-TOPSIS Approach and Its Application to Personnel Selection: A New Trend in Brain Processing and Analysis. IEEE Access, 2019; 7, pp. 29734-29744.
8. Levine, N. Generalized closed sets in topological spaces. Rend. Circ. Mat. Palermo., 1970; 19, pp. 89–96.
9. Coker, D. An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 1997; 88, pp. 81–89.
10. Salama, A.A.; Alblowi, S.A. Neutrosophic set and neutrosophic topological spaces. IOSR Journal of Mathematics, 2012; 3(4), pp. 31-35.
11. Abdel-Basset, M.; Chang, V.; Gamal, A. Evaluation of the green supply chain management practices: A novel neutrosophic approach. Computers in Industry, 2019; 108,pp. 210-220.